

FSTY-405 — Lab 9

Optimization

Objectives

Silvicultural decision analysis. Finding an optimal stand density management prescription using TADAM's Excel add-in and the Excel *Solver*. Useful spreadsheet tools and techniques.

Modelling tending operations, economics

We shall find the most profitable combination of establishment density, timing and intensity of thinnings, and clearcut age in a lodgepole pine stand. These are our *decision variables*. The optimality criterion (*objective function*) will be the *Site Value* (SV), also known as *Land Expectation Value*.

Initial density

We start the growth projection at breast height with a certain number of surviving trees per hectare (one of the decision variables). For planting, assume a fixed cost per hectare plus a direct cost per tree:

$$p_{fixed} + p_{direct} * N , \tag{1}$$

where N is the number of trees surviving to breast height. Make $p_{fixed} = 500$ \$/ha and $p_{direct} = 0.01$ \$/tree. We shall arrange the spreadsheet so that values of parameters such as these can be easily changed.

Costs and revenues that are not significantly affected by changes in the decision variables can be ignored for optimization purposes. For instance, administration costs, pest and fire control, surveys, etc. All that happens is that the SV will be incomplete, and cannot be used as an absolute measure of profitability for the forest operation. Here we are only interested in the part that changes with the decisions.

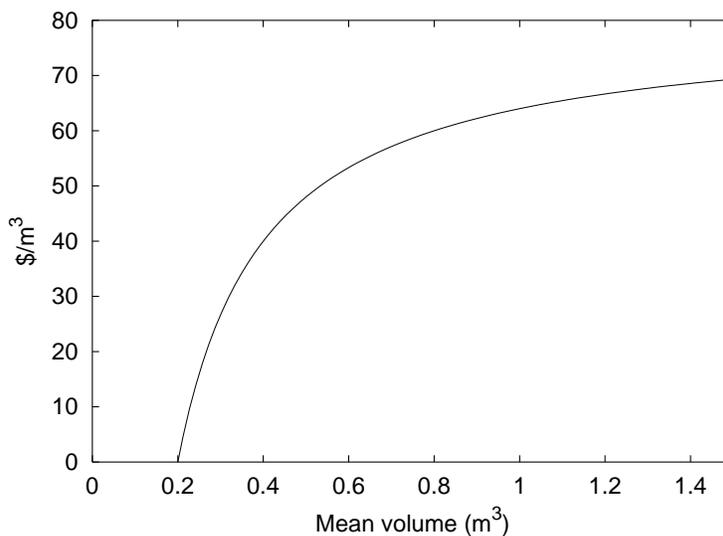
Harvest

At the clearcutting age, there is a harvesting (logging) cost and a timber revenue. We use fairly simple relationships for these here. Logging cost is assumed to be made of a fixed per hectare plus a direct cost per tree:

$$h_{fixed} + h_{direct} * N , \quad (2)$$

with $h_{fixed} = 10$, $h_{direct} = 0.01$.

The revenue per cubic metre at the stump varies with tree size, reflecting mill conversion factors and possibly yarding/loading costs, etc., according to the following figure.



The equation is

$$asympt * \left(1 - \frac{vzero}{v}\right) ,$$

with $asympt = 80$, the *asymptotic* value, and $vzero = 0.2$, the tree volume for zero revenue. Multiplying by the total volume V we get the value per hectare:

$$asympt * (V - vzero * N) . \quad (3)$$

Again, we shall make the values of $asympt$ and $vzero$ easy to change in the spreadsheet. In a real situation it would be easy to use other relationships, based on any or all of the state variables.

Thinning

To keep things simple, we assume that the thinning cost is 30% higher than (2) above, and use (3) for the revenue. Of course, here N and V are the trees per hectare and total volume removed in the thinning. If the revenue is not positive we take it as zero, that is, we leave the trees on the ground (pre-commercial, thinning to waste). Therefore, the thinning revenue equation is actually

$$\max\{0, asympt * (V - vzero * N)\} \quad (4)$$

(use the MAX function in Excel, or better, the POSITIVE function explained on page 8).

Discounting

To make costs and revenues incurred at different times comparable, they have to be discounted or compounded to a common date. We will discount net revenues from age t to age zero through dividing by $(1 + r)^t$, where r is the discount rate. Start with $r = 0.04$ (4%). The sum of all these discounted net revenues is the net present value (NPV).

The NPV represents the return from one rotation. Faustmann (1849) proposed as a better criterion the total discounted value from an infinite sequence of identical rotations, the *land expectation value* or *site value* (SV). If T is the rotation length, we have

$$SV = NPV + \frac{NPV}{(1 + r)^T} + \frac{NPV}{(1 + r)^{2T}} + \dots$$

This infinite sum turns out to be¹

$$SV = \frac{NPV}{1 - (1 + r)^{-T}}. \quad (5)$$

Implementation, unthinned case

Open Excel. If not already installed (look for the *TadamP* category under *Insert > Function...*), you need to install the TADAM add-in: Download the Excel add-in from <http://forestgrowth.unbc.ca/tadam>. Unzip, and follow the installation instructions in the `readme.txt` file. On the lab computers you are not allowed access to the system directories, so leave the `*.dll` file in your working directory instead.

Set up all parameters, with appropriate labels, somewhere at the top of the spreadsheet: site index, discount rate, *pfixed*, *pdirect*, *hfixed*, *hdirect*, *asympt*, *vzero*. It may be convenient to name these cells (*Insert>Name...*), so that the name can be used in formulas, instead of absolute cell references. Chose a site index of $17 + xy$, where *xy* are the last two digits of your student number.

Then, arrange calculations for the unthinned case roughly like this:

	Age	H	B	N	V	Cost	Return	Net	Discounted
Planting	0	—	—	—	—				
Breast-height		1.3	0		—	—	—		
Harvest									
								NPV:	
								SV:	

Enter you best guess for the decision variables (initial *N* and harvest age). To distinguish the decision variables, fill those cells with some colour. Use another colour for the objective (SV). If you want, colour also the parameter cells (with a different colour).

¹Cunning proof: Note that $SV = NPV + SV(1 + r)^{-T}$. Solve for SV.

Fill-in the appropriate formulas. Use cell references (e.g., names) for the parameters, not their current values. Here is a remainder of the TADAM functions needed:

tdpGrowBA, tdpGrowTPH: The *global transition functions*. Internally, the local transition functions (rates) are integrated with a 4th-order Runge-Kutta method to calculate these.

tdpHeight: Estimates top height from age and site index. Leave the time gain as 0.

tdpAge: Inverse of the previous one; estimates age from top height and site index.

tdpTPHAfter: Estimates the trees per hectare from the basal area after a “typical” thinning.

tdpVolume: Total volume, estimated from the current state variables.

Make a note of the decision variables and SV for your prescription:

.....

Optimization

We could substitute decision variable values by trial-and-error, to find better alternatives (higher SV). Excel includes an add-in, *Solver*, that can do this automatically, usually producing the best possible solution². Similar facilities are available in other spreadsheet software such as Gnumeric and Calc (OpenOffice).

Select *Solver...* in the *Tools* menu. If it does not appear in the menu, you may have to activate it from the *Add-Ins...* entry first. Set the “target cell” (objective, SV), *max*, and the cells to be changed (decision variables). Click *Solve*.

²The name is not quite appropriate. The add-in can be used to solve for variables that cause another one to take a given value, but more importantly, it includes sophisticated optimization algorithms to find maxima or minima subject to various constraints.

Depending on the starting values, the automatic search may take the model out of range, producing nonsensical results or calculation errors. If this happens, try again adding appropriate constraints on the decision variables, for instance, $N \geq 100$.

Result: Does it seem reasonable? How does it compare with your guess?

Note: Under some circumstances, optimization algorithms can fail. The number of iterations may exceed the given limit (*Options* button in *Solve*), and they can only find *local optima*. A local optimum is a point that is better than all its neighbours, but it might not be the over-all best. A good initial guess helps. It is advisable to try several starting points to (partially) guard against these problems.

For a tutorial and more info on *Solver* and optimization in general, see www.solver.com.

Thinning

Make another table, now including one thinning:

Thin.intensity:		(% of B)							
	Age	H	B	N	V	Cost	Return	Net	Discounted
Planting	0	—	—	—	—				
Breast-height		1.3	0		—	—	—		
Before thin						—	—	—	—
Removed									
After thin						—	—	—	—
Harvest									
								NPV:	
								SV:	

There are two new decision variables: thinning age, and thinning intensity (specified, for instance, as % of basal area). Guess values.

Complete the calculations. Hint: calculate removed values by difference between before and after thinning.

Optimize. Result? Better or worse than no thinning? Write down your recommended prescription:

.....

Homework and observations

Include a second thinning. Better or worse? If better, try a third one.

What happens to the harvest age when maximizing NPV instead of SV? Can you see why?

Change parameter values to others that seem to you more reasonable.

Try “optimistic” and “pessimistic” parameter values. How do the results change? What effect does the discount rate have? This testing for effects of uncertainty in parameter values is a form of *sensitivity analysis*. *Solver* produces also additional sensitivity information, see www.solver.com.

All this is not restricted to purely financial aspects. Non timber, non market and social values could be included. Also constraints imposed for environmental reasons or regulations. Note that the cost of such constraints can be easily calculated, by comparison with the unconstrained solution.

In some situations a stand-level analysis is not sufficient, and forest-level consequences need to be considered also. An irregular distribution of ages, or fluctuations in demand, may cause costs and/or values to be higher in some years than in others. The stand-level results may then be a first approximation used as input to a *forest estate model*. Ideally, the forest-level analysis would produce relative values (*shadow prices*) varying over time, that could be used for a new stand-level analysis. The process could be iterated until a satisfactory over-all solution is found, although currently this is not often done in practice. In principle, growth models embedded into forest estate models would be better, but the technology is not there yet.

Refinements

In going further with this, a couple of changes would be advisable to improve reliability:

1. In general, using non-smooth relationships such as (4) in optimization is a *bad idea*. The algorithm can get stuck at local optima caused by function “corners” (e.g., the point at (0.2, 0) in the figure). A trick that works for rounding the corner is to substitute a function $(x + \sqrt{(x^2 + a^2)})/2$ for $\max\{0, x\}$ in (4), where a is a suitably small constant. Try graphing this for various a 's; it gets as close to the original as we want as $a \rightarrow 0$. Note that $x = 0$ is changed to $a/2$.

We can implement this by creating a user-defined Excel function POSITIVE(x) to be used in place of MAX(0, x) (taking $a = 1$, for instance):

- (a) *Tools > Macro > Visual Basic Editor* (or press *Alt-F11*).
- (b) *Insert > Module*.
- (c) Enter:

```
Function POSITIVE(x)
    POSITIVE = (x + SQR(x^2 + 1)) / 2
End Function
```

- (d) *File > Close and Return to Microsoft Excel*.

2. Using the thinning and harvest times as decision variables can also cause trouble. Often one needs to introduce constraints such as (thinning date) < (harvest date) to get sensible results. Although the Solver can handle general constraints like these, they tend to make convergence slower and less reliable. An alternative is to introduce decision variables for the *differences* in timing, e.g., the time elapsed between the first and second thinnings, and between the last thinning and harvest. We can then make these the decision variables, and if necessary constrain them to be non-negative, which is easier for the Solver.