

# FSTY 405 — Lab 6

## TASS

Name: .....

### Objectives

The full TASS is a complicated model, but we can get the flavor by using some of its relationships in simplified situations. We will:

1. Draw the crown of a “typical” open-grown tree for ages 5, 10, 15, . . . , using *gnuplot*.
2. Using that graph, determine the crown development for an initial density of 1600 stems per hectare, with a thinning down to half the number of trees at age 20. To keep things simple, we assume that all trees are the same size, and ignore variations in spacing.
3. Given the crown development, predict tree volume growth.
4. Calculate stem profiles, and compare those for open-grown trees, and for thinned and unthinned stands.

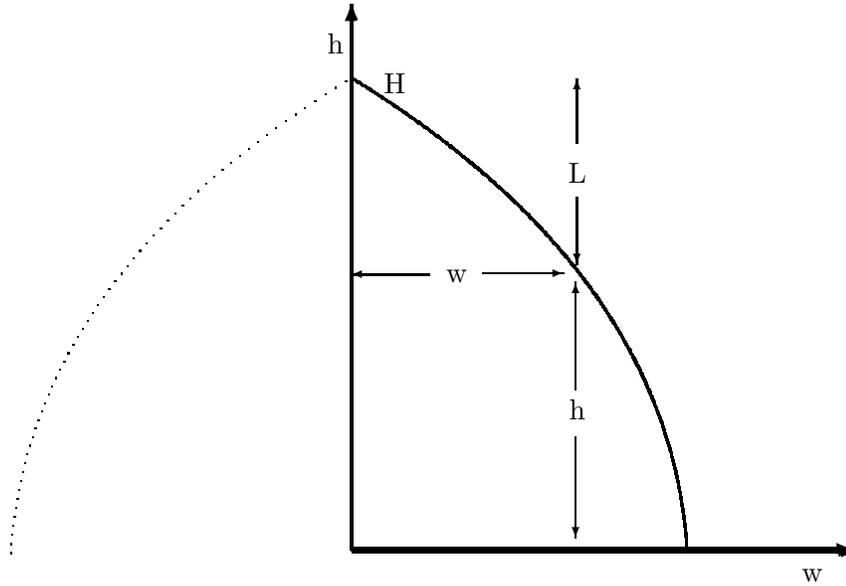
### Crown development

Use the TASS Douglas-fir crown profile model

$$w = 3.43 \ln(L/6.1 + 1) , \tag{1}$$

where  $w$  is crown radius and  $L$  is distance from the tip, both in metres, to plot the crown surface profile at ages 5, 10, 15, . . . , 40 years.

The idea is to work out in *gnuplot* something like this:



with the profiles for the various ages all in the same graph.

First, we need the height of the crown surface,  $h$ , as a function of the horizontal extent  $w$  (for a given  $H$ ),

$$h = H - L .$$

**Solve (1) for  $L$  to get  $L$  as a function of  $w$ , and substitute:**

$$h = H - \dots\dots\dots (2)$$

Now we need to relate  $H$  to the age.

From the 1985 publication by Mitchell and Cameron of yield tables for coastal Douglas-fir generated with TASS, we take the following site index model of Bruce (1981):

$$H = S \exp(b_2[(A + 13.25 - S/6.096)^{b_3} - (63.25 - S/6.096)^{b_3}])$$

$$b_2 = \ln(1.372/S)/[(13.25 - S/6.096)^{b_3} - (63.25 - S/6.096)^{b_3}]$$

$$b_3 = -0.477762 - 0.894427(S/30.48) + 0.793548(S/30.48)^2 - 0.171666(S/30.48)^3$$

$$\text{Years from seed to breast-height} = 13.25 - S/6.096$$

Here  $H$  = top height (m),  $A$  = age at breast height (years), and  $S$  = site index (m at breast-height age 50).

**Homework:** Verify that for site index 28 the above simplifies to

$$H = 69.76 \exp(-20.38/t^{0.7628}), \quad (3)$$

where  $t$  is age from seed.

Assume site index 28 (about average for coastal Douglas-fir). According to Figure 9 in Mitchell (1975), the mean height is roughly 70% of the top height, so for our typical tree we shall reduce (3) by this amount. In *gnuplot* you can use (2) to define

$$h(w,t) = H(t) - \dots$$

with

$$H(t) = \dots$$

given by 0.7 times equation (3).

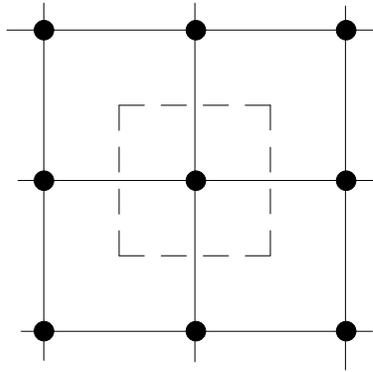
Plot the (half-)crowns for  $t = 5, 10, 15, \dots, 40$ . Use `set grid`. **Print out the result, and attach.**

Include your name in the heading of all graphs.

## Competition

Assume now an initial square spacing of 2.5 metres. If all trees are identical and regularly spaced, the crown can reach up to 1.25 m along rows and columns. Somewhat more in other directions.

From the following diagram (how much area corresponds to each tree?), the number of trees per hectare is  $N = 10000/2.5^2 = 1600$ .



The general relationship between average square spacing,  $s$  (metres), and trees per hectare,  $N$ , is:

$$s = \dots\dots\dots$$

Optional exercise (for home): Work out the analogous relationship for triangular spacing. The resulting hexagonal growing space is more symmetric than the square, with 1.61 m available to each tree along the “spokes”.

To get some sort of average growing space radius  $R$ , we use a mean circular growing space per tree:

$$\pi R^2 = 10000/N \quad \longrightarrow \quad R = \sqrt{\frac{10000}{\pi N}} \approx 1.4\text{metres}$$

At age 20 we perform a thinning, taking out half of the trees. Assume equal-sized trees, and ignore any possible mortality and loss of lower foliage through browsing, etc. Enter the values of  $N$  in the table on page 9.

On the previous graph, **show the extent of the crown** at ages 5, 10, 15, . . . , 40 (you may imagine a competing tree on the right). TASS assumes that needles persist for 5 years. Use pencil/pen cross-hatching and tracing of the foliage boundaries.

Reading from the graph, or by calculation with the equations, **tabulate the actual and potential crown radius, and the height of the base of the green crown** (first columns in the table on page 9).

Hints:

The crown dimensions may be read from the graph, or better, calculated as follows (good idea to check against the graph anyway). The open-grown crown radius  $R_{\max}$  can be obtained from equation (1) for  $L = H$ , e. g., with the `print` command of *gnuplot*. The actual radius  $R$  is the minimum between this and the space available, either 1.4 m or the spacing after thinning **(which is . . . . . m)**.

The height to the base of the crown,  $h_c$ , is easily obtained with the previously defined  $\mathbf{h}(w, t)$  function, with  $w$  equal to the current spacing and  $t$  equal to the age minus 5. Obviously, negative values mean that the crown runs to the ground.

The total height  $H$  can be obtained from the  $\mathbf{H}(t)$  function already available.

## Volume growth

TASS estimates the annual volume increment of a tree (for Coastal Douglas-fir), in cubic metres per year, as

$$dV = 0.0001606F^{1.133}[1 - \ln(F/F_{\max})]^{1.193}, \quad (4)$$

where  $F$  is a weighted foliage volume, and  $F_{\max}$  is the potential  $F$  without competition<sup>1</sup>. These are calculated using photosynthetic efficiency and leaf retention weights of 1.00, 0.86, 0.75, 0.63, and 0.40, for the volumes of 1 to 5 year-old needles, respectively.

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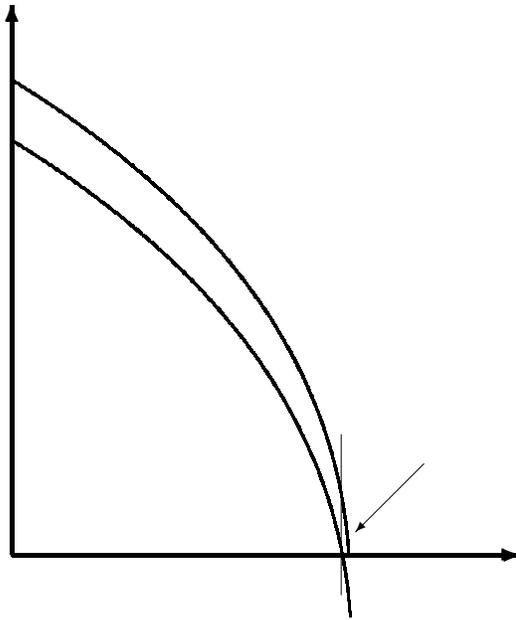
<sup>1</sup>The coefficients are more recent estimates, slightly different from those in the 1975 publication (Di Lucca, pers. comm.)

The annual foliage sheaths have a uniform vertical depth, because the crown surface preserves its shape, just lifting following tree height growth. Ignoring the small differences in height increment and crown width within a 5-year period, the foliage volume is simply

$$F = 0.728C \Delta H ,$$

where  $C$  is the horizontally projected crown area, and  $\Delta H$  is the total height increment of the last 5 years.

Note: Actually, this is exact only if the base of the crown is above the ground. Otherwise, for open-growing trees, there is an underestimation:



The difference, however, is only likely to be important for small trees.

**Explain where did the 0.728 come from, and check:**

.....  
 .....  
 .....  
 .....

Substituting in (4) for  $F$ , and multiplying by 5, the five-year tree volume increment should therefore be well approximated by

$$\Delta V = 0.0005604(C \Delta H)^{1.133}[1 - \ln(C/C_{\max})]^{1.193},$$

or, for circular crowns of area  $C = \pi R^2$ ,

$$\Delta V = 0.002050(R^2 \Delta H)^{1.133}[1 - 2 \ln(R/R_{\max})]^{1.193}. \quad (5)$$

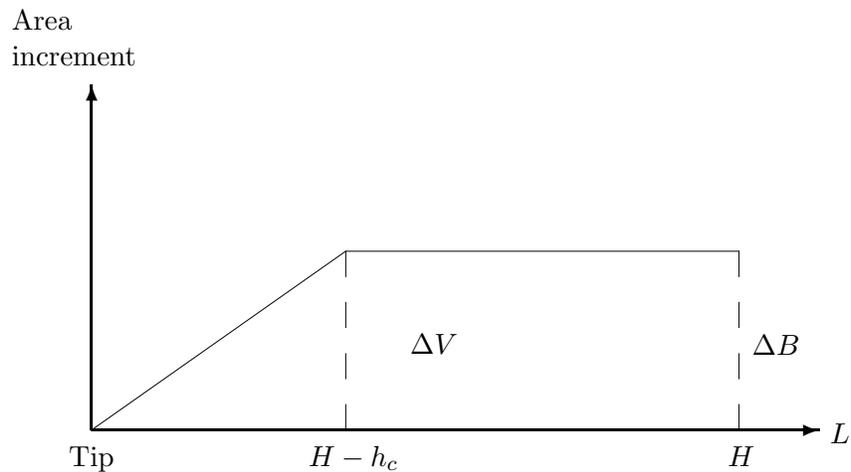
**Homework:** Verify, and make sure that you understand all this.

**Enter in the table (page 9) the 5-year volume increments, the accumulated tree volume, and the volume per hectare.** Note that you should place the last two values in-between consecutive table rows.

Hints: You need  $\Delta H$ , so first fill-in columns for  $H$  and  $\Delta H$ . You can use the `print` command and the function already defined in *gnuplot*: `pr H(5)`, `pr H(10)`, ... For the volume increments, you could use also a *gnuplot* function: `DV(R,Rmax,DH) = 0.00205*(...`

## Stem profile

TASS uses Pressler's "law" to grow the whole bole profile. According to this, the increment in cross-sectional area at any point is proportional to the length of live crown above that point:



The total inside area is the volume increment  $\Delta V$  (five-year increment). This is related to the tree total height  $H$ , crown base height  $h_c$ , and tree basal area increment  $\Delta B$ :

$$\Delta V =$$

**(complete the right-hand side, it should equal the area of the triangle plus the area of the rectangle in the figure).**

Therefore,

$$\Delta B = \frac{2\Delta V}{H + h_c}. \quad (6)$$

In addition, the cross-sectional area increment  $\Delta b$  at any distance  $L$  from the tip can be easily calculated (see the figure):

$$\Delta b = \min\left\{\frac{\Delta B}{H - h_c}L, \Delta B\right\}.$$

**Homework:**

- (a) Given the volume increments from the previous problem, calculate (e.g., using Excel) cross-sectional area increments for each age interval and for points spaced at 0.5 m intervals along the stem. Accumulate these increments over time, and graph the stem cross-sectional area profiles for the various ages.
- (b) Obtain the diameters from the areas, and graph the diameter profiles.
- (c) Do the same for an open-grown tree, and for a stand planted at 1600 sph and left unthinned. Compare the differences in stem taper due to stand density.
- (d) How would you evaluate the effects of pruning on growth and on tree form?

**Table:**

$t$	$N$	$R$	$R_{\max}$	$h_c$	$H$	$\Delta H$	$\Delta V$	$V$	Vol/ha
5									
10									
15									
20									
25									
30									
35									
40									