

FSTY-405 — Lab 4

Visual Modelling

Name:



Objectives

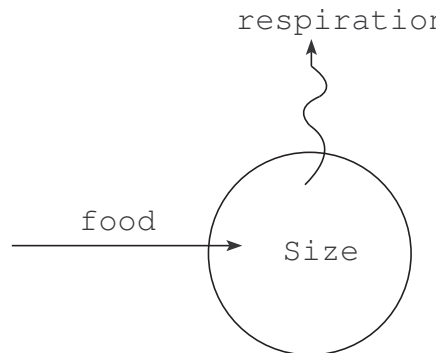
To examine some basic dynamical systems ideas. To learn about visual simulation software.

We will model a very simple artificial system. It shows key aspects of tree growth without the complications of a more realistic model.

Vensim, Powersim, Simile/AME, and Stella are general-purpose simulation packages based on Forrester's "System Dynamics" notation, also called *stock-and-flow* or *rate-and-level* diagrams. Free versions of the first three can be downloaded from the internet; UNBC has a site license for Stella. All these descend from Dynamo, which is more text- and less graphical-oriented. We will use Vensim.

Simulating The Blob

The Blob, as in the old horror movie, is a "thing" that grows by gobbling up food, including people. It also loses matter, burnt in maintenance respiration:



1. We describe **the state** of the system by the blob's "size". Let us use its biomass, call it W . Could have used the area instead, which is proportional to W , or its diameter, etc. These are equivalent state descriptions because they are directly related, e.g., $A \propto D^2$ assuming a flat disk. The symbol " \propto " means "proportional to".

Start Vensim. Explore the menus and tools (you get a description by pointing to an item for a second or two).

Menu: **File** > **New Model...** (or click the *New Model* button), change time units to Minute. If you need help, can look up Chp.5 in **Help** > **User Guide**.

With the *Box Variable-Level* tool, create

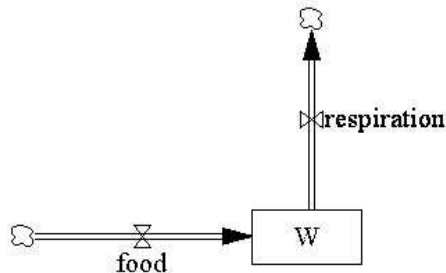


near the center of the screen (type *W*, *Enter*).

This is our state variable, aka *level* or *stock*.

2. How does *W* **change** with time? It increases through food intake, decreases through respiration losses.

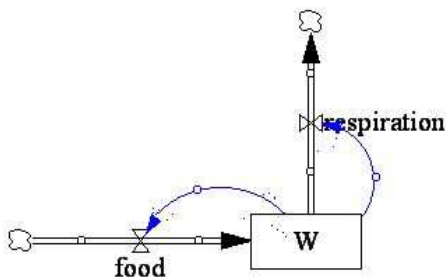
Select the *Rate* tool. Click somewhere to the left of the *W* box, and then inside the box; type “food” and press *Enter*. This represents the food intake, a flow from the outside world into the box. Add the respiration losses:



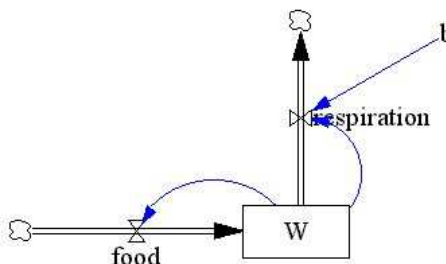
We have a box with a *stock* of material (biomass), and *flows* of material going in and out. The “pipes” with arrows represent flows, controlled by a valve or tap. The little clouds represent the outside world.

More generally, one may think of the *level* of a state variable (*W*), and *rates* of change of it (state variables may or may not represent a quantity of material, e.g. diameter). Food intake and respiration rates are changes of *W* per unit of time, e.g., in tonnes/hour.

3. **Rate equations.** Both rates depend on the current size. With the *Arrow* tool click on the box, and then on the taps. Shape the arrows by dragging the little circles on them (alternatively, you can click on the box, then on an intermediate point, and finally on the tap):



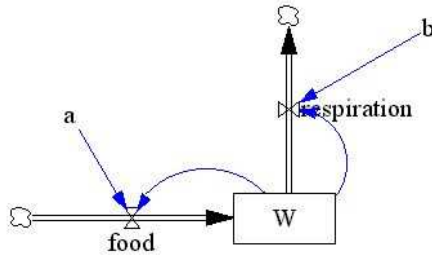
Assume that the respiration rate is proportional to W , that is, $\text{respiration} = bW$, where b is some constant. Represent this constant using the *Variable-Auxilliary/Constant* tool, and show that the rate depends on it:



What is the actual relationship? Select the *Equation* tool. Click on *respiration*. Note that b and W appear as variables influencing the rate. Enter $b*W$ in the equation box.

Now the food. Assume that the food intake rate is proportional to the blob's perimeter P . If the blob is close to a flat disk, we know that $P = \pi D$, $A = (\pi/4)D^2$, and that W is proportional to A . Then, $W \propto D^2 \propto P^2 \propto \text{food}^2$. Therefore, $\text{food} \propto \sqrt{W}$, or $\text{food} = a\sqrt{W}$ for some constant a . Think about it!

Add a to the diagram, and enter $a*\text{SQRT}(W)$ for the food intake rate:



Almost done! With the *Equation* tool, give values to a and b (say 1, for starters).

4. We are left with the W box in black, indicating something missing. To run a simulation we need to give it a starting (or initial) value. Start with a very small blob, say 0.01 tonnes (10 kg). With the *Equation* tool, set this **initial value**.

By the way, note that the equation for W says “ $W = \text{INTEG}(\text{food} - \text{respiration})$ ”. This means that the size W is the accumulation (integral) of the food - respiration flows (or rates).

If you want, you can specify units in the equation dialogs. That can be useful in labelling graphs, and for automatic model checking, among other things.

5. All set, now on with **the simulation**. *Set*, *Run* (buttons on the top toolbar). Double-click on W to select it as the main variable of interest (the “Workbench variable”). Click on the *Graph* tool, left side toolbar.

Is growth stopping (maxing out) too soon? Let’s reduce the respiration losses: change b to 0.1 and try again (choose to overwrite the data set).

Change a and b to other values, and see what happens (use different data set names so that you can graph the results together). Try the *Causes Strip* tool. Simulation runs can be included or excluded from the graphs by moving them between loaded and available info in the *Datasets* tab of the *Control Panel*.

6. **Other outputs.** We might be interested in how the Blob’s diameter changes. Add the output variable D somewhere to the right of the W box, using the *Variable-Auxilliary/Constant* tool. Add an arrow from W to D .

Assume that the Blob is about 10 cm high, with a density of about 1 (human flesh). Then, one m² contains 10 × 10 = 100 litres, weighting 0.1 tonnes. Therefore, for D in metres, $W = 0.1(\pi/4)D^2$, from where, $D = 3.57\sqrt{W}$, approx. Think!

Enter this equation for D (*Equation tool*). Might as well set the units to metres.

Set, Run. Double-clicking on D and selecting the *Causes Strip* plots both D and W .

7. Experiments

- (a) What happens if there are no respiration losses ($b = 0$)?

Result? Why?

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- (b) Cut off the food to a 70 tonnes Blob (initial $W = 70$, $a = 0$).

Result? Why?

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- (c) Experiment with various parameter values (a 's and b 's).

Result? Why?

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- (d) See what effect, if any, has changing the step size (**Model > Settings...**), and/or the integration method (*Euler* or *RK4* on the left when pressing *Set*).

Result? Why?

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Discussion

Cute, but what is going on? How to tell what various a 's and b 's are going to do? Need some math!

The transition function

The rate is given by the so-called *local transition function*

$$\begin{aligned} \text{growth rate} &= \text{food intake} - \text{respiration} \\ \frac{dW}{dt} &= a\sqrt{W} - bW \end{aligned}$$

The respiration loss grows faster than the food intake, so that the growth rate slows down. When losses finally (almost) equal gains, growth becomes zero (stops):

$$a\sqrt{W} - bW = 0 .$$

When does this happen? Solve for W . One way:

$$\begin{aligned} \text{divide by } \sqrt{W} : \quad a - b\sqrt{W} &= 0 \\ \sqrt{W} &= \frac{a}{b} \\ W &= \left(\frac{a}{b}\right)^2 , \end{aligned}$$

i.e., growth stops when W approaches $(a/b)^2$. This is the curve's *asymptote*. Check against your simulations.

It might be handy to *reparameterize*, using the asymptote $\alpha \equiv (a/b)^2$ as a more “meaningful” parameter. E.g., by substituting $a = b\sqrt{\alpha}$:

$$\frac{dW}{dt} = b\sqrt{\alpha}\sqrt{W} - bW = b(\sqrt{\alpha W} - W) .$$

Note that b determines how fast the process develops. Verify by simulation.

Step size and integration

Calculations are done at time intervals of the specified size, here one minute. The Euler method simply uses the equation to calculate the W -increment over the interval, given W at the start of it. Then adds the increment to the starting value, and repeats the process.

This can be interpreted in two ways. One, as an approximation to the true slope (derivative). Or the model might have been formulated and fitted with data for that time interval size (a *finite differences equation*, or simply *difference equation*, instead of a *differential equation*).

Either way, the results can vary somewhat with the step length used (the Euler method approximates the periodic increment by the initial slope). For small enough steps the approximation would be OK. The fourth-order Runge-Kutta method (RK4) uses a more complicated procedure, combining derivatives calculated at both extremes and at the center of the interval. It is one of many methods that give more accuracy than Euler's for a given step size, at the cost of more computation per step.

Changing state variables

Back to the Blob. Obviously, it can be somewhat dangerous attempting to find its weight. On the other hand, we can easily get reasonable diameter measurements on photographs taken from a helicopter. It might be better, then, to produce a model using D instead of W as the state variable. W can be calculated from D as an output, knowing that $W = (\pi/40)D^2$.

Substitute this into the transition equation for W :

$$\begin{aligned}\frac{dW}{dt} &= a\sqrt{W} - bW \\ \frac{d}{dt}\left(\frac{\pi}{40}D^2\right) &= a\sqrt{\frac{\pi}{40}D^2} - b\frac{\pi}{40}D^2\end{aligned}$$

$$\frac{\pi}{40} \cdot 2D \frac{dD}{dt} = a\sqrt{\frac{\pi}{40}}D - b\frac{\pi}{40}D^2$$

$$\frac{dD}{dt} = \frac{a}{2}\sqrt{\frac{40}{\pi}} - \frac{b}{2}D.$$

The initial values D_0 and W_0 are related by $D_0 = \sqrt{(40/\pi)W_0}$.

Implement this in Vensim, and verify that you get the same results as before.

What is the maximum diameter?

(Hint: see how maximum W was found on page 7).

Being able to carry out manipulations like this is one of the advantages of differential equations over difference equations. Can be done with differences, but the results are a lot messier.

Real-time simulations

Newer versions of Vensim allow changing parameter values through sliders, observing outputs in real-time. For this, click the button “Automatically Simulate a Change”, instead of “Run a Simulation” (located immediately to the right of this last one). Change parameter values by dragging the sliders or typing a number. Small output graphs are shown for state and rate variables.

To get a reasonable range on the sliders, you should set appropriate values for *Minimum Value*, *Maximum Value*, and *Increment* in the equation editing dialog for the parameters. You would also want to display proper output graphs and/or tables: use the *Input Output Object* tool, selecting *Output Workbench Tool* for Object Type, and Graph or Table in the drop-down list.