Evaluating Passive Microwave Radiometry for the Dynamical Transition From Dry to Wet Snowpacks

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Abstract—The microwave response of an idealized snowpack is evaluated for a change of the liquid water content (LWC) ranging from 0.0 m$^3$ m$^{-3}$ to 0.1 m$^2$ m$^{-3}$, a realistic range for a dry to a wet snowpack transition. Next, the microwave radiometric behavior of a snowpack as a function of LWC is investigated using a coupled snow hydrology-microwave-emission model that consists of a direct combination of multi-layer snow hydrology model and a forward model of microwave emission based on the simulated multi-layer snowpack. The microwave response during the transition is characterized by an initial increase of the brightness temperature ($T_b$) followed by a monotonic attenuation of $T_b$ with a linear increase of LWC. Thus, the microwave response $T_b$ to the increase in LWC exhibits a convex shape. The early amplification of $T_b$ is caused by a sharp increase of the absorption coefficient attributable by a relatively small amount of LWC within the snowpack. This peak of $T_b$ can be explained by the decrease in layer reflectivity and transmissivity in the Microwave Emission Model of Layered Snowpacks. The decrease of snow layer transmissivity indicates that the snowpack operates as an opaque medium in the microwave spectrum for small values of LWC. However, when LWC continues to increase, the increase of interface reflectivity at the atmosphere-snowpack interface begins to suppress the increasing $T_b$ trend. As a result, the $T_b$ subsequently decreases because the interface transmissivity, a complement of reflectivity, decreases asymptotically. This arched (convex) behavior of the $T_b$ signal with increasing LWC is also exhibited by the Special Sensor Microwave Imager and the ground-based microwave radiometer observations concurrent with the presence of LWC in the simulated snowpack. For the validation of the $T_b$ response to the LWC, simulations using a coupled snow hydrology-microwave-emission model are compared against both passive microwave satellite and ground-based radiometer observations during the 2002–2003 Cold Land Processes Experiment (CLPX). Two Meso-cell Study Areas from CLPX are selected to evaluate the coupled model performances of $T_b$ and snowpack physical properties in response to changes in LWC.

Index Terms—Brightness temperature ($T_b$), liquid water content (LWC), microwave signature, snow model.
I. INTRODUCTION

THE LIQUID water content \( (LWC \ [m^3 \cdot m^{-3}] ) \) of snowpacks has been linked to the development of critical structural instabilities leading to avalanche risk and possible loss of life [1]–[3]. This is particularly important in mountainous regions such as western Canada and the United States where abundant snowfall and a climate moderated by the Pacific Ocean can lead to unstable snow layers that may trigger avalanches. Therefore, it is critical to better understand and monitor snowpack properties to predict avalanche risks for highway infrastructure, ski operations, and other outdoor recreational activities [4]–[7] in regions such as western North America. In a temperature-controlled test, [8] found that liquid water in snow layers significantly affects the stability of the snow structure. From this laboratory test, the maximum amount of \( LWC \) prior to instability was shown to be \( 0.013 \, m^3 \cdot m^{-3} \) in wet snow conditions. This suggests that a small increase in volumetric liquid water content can alter snow conditions during the transition from dry to wet snow regimes. Various models have been used to estimate and understand the complex behavior of liquid water within a snowpack (e.g., SNOWPACK [9], SNTHERM [10], and CROCUS [11] at local scales, and the GISS snow model [12] at the global scale). While snow density and depth are relatively well simulated in scales, and the GISS snow model [12] at the global scale). While snow density and depth are relatively well simulated in these models, it is still difficult to estimate the snowpack \( LWC \). It is also challenging to directly measure the \( LWC \) in situ using various monitoring methods such as snow core sampling, time domain reflectometry [13] and ground penetrating radar [14].

The most detrimental aspect of measuring \( LWC \) in a natural snowpack is the destructive nature of these methods, leading to snowpack disturbance when measured. In addition, in remote regions and areas of complex terrain, access to field sites can be challenging and in situ measurements dangerous to conduct.

Unlike other snowpack monitoring methods, microwave radiometry provides a non-destructive way to estimate the snowpack \( LWC \) [15]–[17]. To first order, the observed brightness temperature \( (T_b) \) from a satellite or an airborne radiometer can be estimated from the product between the earth’s surface temperature and snowpack emissivity when snow is present. When the snowpack is ripe and wet, the emissivity becomes large compared to that of a fresh and dry snowpack. Therefore, the microwave emissions from the wet (old) snow are more attenuated than those from a dry (new) snowpack. Consequently, changes in snowpack physical properties are related to temporal changes in snowpack emissivity and \( T_b \). In this paper, the relationship between snow radiometric variables and snowpack physical properties is investigated using a forward microwave emission model coupled to a snow physics model.

Based on theoretical research and laboratory experiments [18]–[20], various forward models to simulate \( T_b \) based on multi-layer snowpacks have been developed by [21]–[23]. The central objective of forward models is to determine the real and imaginary parts of permittivity of snow layers based on microwave transmission theory. The real and imaginary parts of snowpack permittivity can be subsequently related to physical properties such as snow density, snow grain size, and \( LWC \). The radiometric response of the wet snowpack is much more complex than that of the dry snowpack based on extensive characterization studies that have been conducted in the past [24]–[26]. This is due to the fact that the real and imaginary parts of the permittivity are significantly attenuated by the presence of liquid water within the snowpack. Water molecules are more effective in attenuating the microwave radiation as compared to other constituents in the snowpack such as air, ice, and water vapor. Internal melting of the snowpack also provides uncertainty in determining the complex permittivity behavior of a snowpack as it is a porous medium. For example, [27] found that there is a sharp increase of \( T_b \) with the presence of \( LWC \) in a snowpack during an idealized experiment. Based on this finding, they provided an algorithm for early detection of sub-arctic snowmelt from microwave radiometer data based on rapid changes in \( T_b \) that mark the onset of snowmelt. On the other hand, [28] found a monotonic decrease of \( T_b \), which is an attenuation of energy of the microwave radiation emitted by the snowpack with liquid water content. In summary, there is a sharp increase in \( T_b \) with \( LWC \) initially, followed by a monotonic decrease of the \( T_b \) after a certain threshold of \( LWC \) is reached. Here, it is also noted that experimentally [27] found a sharp increase of \( T_b \) and successive decreases with increasing \( LWC \) from snow measurements from ground-based sensors. Although [29] and [30] evaluated microwave radiometry with snow forward modeling and measurements, a theoretical evaluation of \( T_b \) sensitivity to \( LWC \) in the transition from dry to wet snow has not been conducted. Whereas there are some examples of previous studies focused on wet snow [27], [28], a systematic study of radiometric behavior in the dry-to-wet transition in terms of \( T_b \) sensitivity to \( LWC \) is lacking in the literature.

In this paper, we address how the radiometric behavior of snow responds to the presence of \( LWC \) within the snowpack using a coupled multi-layered snow hydrology-microwave emission model (hereafter, coupled model). That is, the objective of this study is to investigate how the \( T_b \) of a snowpack responds to the \( LWC \). To evaluate the theoretical radiometric behavior of the snowpack, a microwave forward model, an adapted form of the Microwave Emission Model of Multi-layered Snowpack (MEMLS) [22], [23], is used. MEMLS also provides estimates of the real and imaginary parts of permittivity as well as the absorption and scattering coefficients of each snow layer. Other radiative transfer parameters within the snowpack including transmissivity, reflectivity, and emissivity can also be estimated by MEMLS, which can be used to explain the processes affecting \( T_b \) response induced by a change in \( LWC \).

The paper begins with an introduction on the two stages of a snowpack response due to the presence of \( LWC \) [27], [28],
followed by a discussion of $T_b$ sensitivity to $LWC$ in a single-layered, idealized snowpack using MEMLS. Next, the decomposition of $T_b$ into an interface transmissivity, $(1 - s_n)$, and the snowpack outward brightness temperature, $D_n$, is introduced. Subsequently, the evolution of $(1 - s_n)$ and $D_n$ as a function of subsidiary variables such as complex permittivity, absorption, scattering coefficients, and energy transfer variables, i.e., $r$, $t$, and $e$ is discussed. This additional background section relates the radiometric variables to the physical properties of the snowpack, with a focus on $LWC$. Then, a coupled snow hydrology-microwave emission model is used to validate the $T_b$ behavior in response to $LWC$ changes. In the application section, data from the Cold Land Processes Field Experiment 2002 (CLPX) [31] are used to force the coupled model and to validate the $T_b$ simulation against observations from the Special Sensor for Microwave Imager (SSM/I). The coupled model was applied to two distinct sites, and the outcomes are reported in the Results section. In addition to SSM/I, a data set from a ground-based microwave radiometer (GBMR-7) [32] is also compared with the multi-layered coupled model at the point scale. Finally, the Conclusions section discusses other research needs regarding the applications of a coupled snow hydrology-microwave emission model.

II. Background

The detection of $LWC$ using microwave radiometer data such as the Scanning Multi-channel Microwave Radiometer (SMMR) and SSM/I from 1979 to 2001 in arctic and subarctic areas was demonstrated by [27]. They listed three reasons for the application of microwave remote sensing to detect snowmelt: 1) microwave emission is sensitive to the presence of water in the snowpack because the complex permittivity of liquid water is much higher than that of ice and snow; 2) data collection is not significantly influenced by cloud cover due to the long microwave wavelengths, and 3) a relatively continuous and global scale microwave data set since 1978 to the present is available from SMMR, SSM/I, and more recently the Advanced Microwave Scanning Radiometer. Künnzi et al. [33] showed that the difference between 18 GHz and 37 GHz $T_b$s varies from positive values in winter to negative values in spring, reflecting seasonal changes in snowpack conditions, namely a seasonal snowpack warming and increase in $LWC$. Then, [34] proposed an algorithm to detect the onset of snowmelt when the $T_b$ increases sharply due to the presence of liquid water in the snowpack. This was the first exploration of snowmelt detection using SMMR and SSM/I microwave radiometer data sets including spatial mapping of the arctic and subarctic areas.

In the Swiss Alps, [28] relied on a combination of a radiometer, radar, and observational methods to classify snow conditions as dry, wet, old, and/or new. They found that $T_b$ decreases when $LWC$ increases through melt, or during rain on snow events. This observation in the Swiss Alps is opposite to the sharp increase of $T_b$ caused by the $LWC$ increase reported in earlier studies. The seasonal variation of wet snow $T_b$ is unexpected because of the shorter microwave penetration depths for wet rather than dry snow. The radiometric characteristics of wet snow were investigated by others [17], [35], and specifically, [36] reported that the dielectric behavior of wet snow must be modeled as an unstable thermodynamic system. Thus, the estimation of complex permittivity and attenuation constants in wet snow is much more complex than in the dry snow case. Although the real part of the permittivity and the attenuation constant both increase with $LWC$, the imaginary part of permittivity exhibits a more complex behavior. Even with a nonlinear behavior of the imaginary part of permittivity, the overall $T_b$ exhibits a decreasing trend when $LWC$ is significantly introduced in a snowpack.

Next, a scope on how the microwave response of a snowpack is simulated using MEMLS is presented. Although a multi-layer snow hydrology model describes more realistic snow hydrological processes, a single-layer model is first used here to identify unambiguously the contribution of $LWC$ to the radiometric response of the snowpack. Then, a multi-layer model is used to identify the specific radiation transfer mechanisms affected by changes in $LWC$ that alter $T_b$. The coupled model with a multi-layer snow scheme is used in Section IV to validate the findings from the idealized simulation of a single-layer snowpack. Sections II-A–II-E below describe how the idealized model is set up and applied to evaluate the $T_b$ responses to $LWC$.

A. Dielectric Permittivity

The real and imaginary parts of permittivity are determined from snowpack physical and hydrological properties such as snow temperature, density, grain size, and $LWC$. From these quantities, the layer reflectivity ($r$), transmissivity ($t$), and emissivity ($e$) are then calculated, and the internal upward and downward brightness temperatures ($A_j$, $B_j$, $C_j$, and $D_j$) in the MEMLS equation for $T_b$ are estimated (Fig. 1). Note that MEMLS also considers the interface layer between internal snow layers. To calculate the reflectivity ($s_n$) of this interface layer, the Fresnel coefficient ($F_n$) determined by the real and imaginary permittivity of the snowpack is used. To investigate the snow hydrological processes used here, we start by characterizing the “snow physical properties” and then move to “radiometric variables” (bottom-to-top direction in Fig. 1). The two are linked by the “extinction coefficient and permittivity” estimation. Changes in $LWC$ within the snowpack, indicated by a circle on the bottom box in Fig. 1, lead to changes in the absorption coefficient, $\gamma$, and the real $\varepsilon'$ and imaginary $\varepsilon''$ parts of the permittivity, as well as $F_n$. These variables are specified by triangles in the middle box of the figure. Wiessmann and Mätzler [22, eqs. (53)–(57)], [23] express the wet snow permittivity as a function of $LWC$. Among them, the Debye parameters are determined as a function of the static $\varepsilon_{sw}$ and infinite $\varepsilon_{\infty w}$, permittivity of the wet snowpack

$$\varepsilon_{sk} = \frac{LWC}{3} \frac{\varepsilon_{sw} - \varepsilon_d}{1 + A_k(\varepsilon_{sw}/\varepsilon_d - 1)}$$

$$\varepsilon_{\infty k} = \frac{LWC}{3} \frac{\varepsilon_{\infty w} - \varepsilon_d}{1 + A_k(\varepsilon_{\infty w}/\varepsilon_d - 1)}$$

where $\varepsilon_{sk}$ is the static permittivity at $k$th polarization factor, $\varepsilon_d$ is the dry snow permittivity, and $A_k$ is the depolarization...
factor. Here, the static and infinite permittivity of wet snow are linearly related to the $LWC$. The terms $e$ and $s_n$ are calculated in the “Microwave transmission” module. Finally, the outward brightness temperature, $D_{n+1}$, equivalent to $Tb$, is obtained from the product between $D_n$ and $(1 - s_n)$. The subscript $n$ is only used to indicate the layer number indexed from bottom to top. The forward simulation of $Tb$ by MEMLS uses snowpack physical properties simulated by the snow hydrology module, and then the decomposition of $Tb$ into $D_n$ and $(1 - s_n)$ can be investigated. Here, it should be noted that the “modeling process” workflow runs in the opposite direction to the “interpretation process.” When modeling the microwave emission, the snow physical properties need to be prepared or simulated, and any forward model can be used to generate the microwave emission properties. On the other hand, the “interpretation process” is defined by a top-down workflow. That is, the snowpack physical properties, here $LWC$, are estimated from the radiometric response using the coupled model.

### B. Brightness Temperature Response to Liquid Water Content

Our analyses begin with the simulation of an idealized, single-layer snowpack using the microwave forward model. Table I summarizes the physical characteristics of the snowpack used for the simulation of $Tb$ response to the $LWC$. The $LWC$ increases from $0.0 \text{ m}^3 \text{ m}^{-3}$ to $0.1 \text{ m}^3 \text{ m}^{-3}$, corresponding to the transition from dry to wet snow regimes. The snow temperature is fixed at 273.15 K assuming that melting occurs in the snowpack during the simulation. The density and depth are determined by considering that the $LWC$ increases as the snow depth decreases due to melt [Fig. 2(a)]. The snow grain size is assumed constant with a diameter of 1 mm and the corresponding correlation length of 0.07 mm to separate the effects of grain size from $LWC$ effects on the $Tb$ response [37].

The goal of this assumption for the constant grain size is aimed at isolating the effect of $LWC$ on $Tb$ in passive microwave remote sensing. Tests with different grain sizes larger than 1 mm did not yield significant differences in the $Tb$ response (not shown). Because $LWC$ more significantly influences the imaginary part of permittivity in snow, thus increasing the absorption coefficient, the role of grain size toward scattering coefficient is negligible.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Description of the Idealized Snowpack Used in Model Simulations of $Tb$ Response Shown in Fig. 2(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>State Variable</td>
<td>Value [unit]</td>
</tr>
<tr>
<td>Snow temperature</td>
<td>273.15 [K]</td>
</tr>
<tr>
<td>Snow density</td>
<td>100-120 [kg m$^{-3}$]</td>
</tr>
<tr>
<td>Snow depth</td>
<td>18-20 [cm]</td>
</tr>
<tr>
<td>Snow Water Equivalent</td>
<td>0.02 [m]</td>
</tr>
<tr>
<td>$LWC$</td>
<td>0.0-0.1 [m$^3$ m$^{-3}$]</td>
</tr>
<tr>
<td>Grain diameter</td>
<td>1 [mm]</td>
</tr>
</tbody>
</table>

Fig. 2(b) shows the simulated response of the $Tb$ at 37.5 GHz and horizontal polarization (h-pol.) to changes in $LWC$. This frequency was selected because its wavelength ($\sim 8$ mm) is consistent with the range of natural snow grain sizes of interest, and its scattering properties and horizontal polarization is more sensitive to the vertical snow structure [38]. There is an initial increase in $Tb$ starting from 190 K and $LWC$ (termed “the saturation phase”), and the $Tb$ decreases monotonically for $LWC \geq 0.005$ m$^3$ m$^{-3}$ (“the energy dampening phase”). The physical processes leading to this response are illustrated in Fig. 2(b). Schematically, the saturation phase develops by a series of events such as: 1) a decrease of $r$ and $t$, 2) an increase of $e$, 3) and as a result, an increase of $D_n$, 4) and an
increase of \( T_b \), which is equal to \( D_n (1 - s_n) \). Note that 4) and 5) in Fig. 2(b) are merged. On the other hand, in the energy dampening phase, the process is structured by: 1) an increase of \( e' \), 2) an increase of \( s_n \), resulting in a decrease of \( (1 - s_n) \), and 3) a decrease of \( T_b \). Here, also note that 2), 3), and 4), 5) in Fig. 2(b) are incorporated. The convex shape of the \( T_b \) response to \( LWC \) is explained by the decomposition of \( T_b \) into two factors: \( D_n \) and \( 1 - s_n \) (see Section II-C). The individual behaviors of \( 1 - s_n \) and \( D_n \) are examined along with subsidiary radiometric variables in Section II-D and E, respectively.

**C. Decomposition of \( T_b \) Into \( D_n \) and \( (1 - s_n) \)**

Based on [22, eq. (7)], \( T_b \) is the product of \( D_n \) and \( (1 - s_n) \). Here, \( D_n \) is the upward brightness temperature at the top layer of the snowpack. For a single layer, \( (1 - s_n) \) is the top interface transmissivity, where \( s_n \) is the top interface reflectivity. It is assumed that the emissivity of the thin interface is negligible. From Fig. 3(a), \( D_n \) with horizontal polarization (h-pol.) increases up to a constant value of 273.15 K, the physical temperature of the snowpack for \( LWC = 0.005 \, \text{m}^3 \, \text{m}^{-3} \). The asymptotic increase of \( D_n \) is caused by the amplification of \( e \) [Fig. 3(b)]. In Fig. 3(b), it should be noted that the range of \( LWC \) is narrowed until 0.01 \, \text{m}^3 \, \text{m}^{-3} \) to highlight changes in \( r, t, \) and \( e \). In other words, \( s_n \) increases with the \( LWC \). Thus \( T_b \) starts to increase but then remains constant after a certain level of \( LWC \) is attained, while \( s_n \) is attenuated and \( T_b \) decreases after the threshold level of \( LWC \) within the snowpack is reached. This begs the question of why \( D_n \) remains at the snowpack physical temperature as \( LWC \) continues to increase up to 0.01 \, \text{m}^3 \, \text{m}^{-3} \)?? Another question is why \( (1 - s_n) \) decreases for \( LWC > 0.01 \, \text{m}^3 \, \text{m}^{-3} \). Section II-D and E address these two questions, respectively.

**D. Why Does \( D_n \) Converge to a Maximum Temperature?**

The first part of the above question can be rephrased as follows: Does \( D_n \) h-pol. converge to a saturated \( T_b \) in response to \( LWC \) increases? To address this question, \( r, t, \) and \( e \) simulated by MEMLS for the idealized snowpack are analyzed.

Wiesmann and Mätzler [22, eq. (10)] provide a solution of \( D_1 \) for the single-layer idealized snowpack, and \( r \) and \( t \) decrease asymptotically with \( LWC \). Consequently, \( e \) increases up to unity near \( LWC = 0.01 \, \text{m}^3 \, \text{m}^{-3} \). This explains why \( D_1 \) increases and maintains a saturated \( T_b \) up to 270 K in the simulation.

After solving the outward and inward brightness temperatures \( A_{n,0} \) to \( D_n \) in Fig. 1) with interface reflectivity, the single-layer solution \( (n = 1) \), \( D_1 \) is defined by (3), shown at the bottom of the page [22].

Note that the subscripts of all the variables indicate the layer number indexed from bottom to top. In a multi-layer snowpack, index 1 means the lowermost layer just above the soil surface while index 0 refers to the soil layer under the single-layer snowpack. By assuming \( T_{sky} = 0 \, \text{K} \), the second term in the numerator and the denominator remain, \( (A_1 \) and \( A_2 \), respectively)

\[ A_1 = t_1 (1 - s_0) T_0 + r_1 (1 - s_1) T_{sky} + e_1 T_1 = t_1 T_0 + e_1 T_1 \]  
\[ A_2 = \left[ 1 - r_1 s_1 - \frac{s_0 s_1 t_1^2}{1 - r_1 s_0} \right] \cong 1 - r_1 s_1. \]

Because \( T_{sky} = 0 \, \text{K} \) and \( s_0 \) is also assumed to be zero, \( t_1 \) and \( e_1 \) determine \( A_1 \). On the other hand, \( A_2 \) also can be approximated by \( (1 - r_1 s_1) \) for the same reason, where it is reasonable to assume that reflectivity of a dry soil surface under the snowpack is negligible. As indicated in Fig. 3(b), \( t_1 \) and \( r_1 \) approach zero asymptotically with the increase of \( LWC \). Therefore, only the product \( e_1 T_1 \) plays a significant role.

\[ D_1 = \frac{t_1 s_0 r_1 (1 - s_0) T_0 + t_1 (1 - s_1) T_{sky} + e_1 T_1}{1 - r_1 s_0} + t_1 (1 - s_0) T_0 + r_1 (1 - s_1) T_{sky} + e_1 T_1 \]

\[ \left[ 1 - r_1 s_1 - \frac{s_0 s_1 t_1^2}{1 - r_1 s_0} \right] \cong 1 - r_1 s_1. \]  

(3)
increasing role in $D_1$. Now, the remaining question is why $t_1$ and $r_1$ tend to zero as $LWC$ increases. To explain the reason of increasing $D_1$, we focus on the solutions for $r_0$ and $t_0$ from [22, eqs. (23) and (24)]. The remaining unknown variables, $r_0$ and $t_0$ are also given by [22, eqs. (17) and (23)]. Because of the asymptotic increase of $\gamma_s$ and $\gamma_t$, $r_0$ and $t_0$ should decrease. As the $LWC$ increases, the contribution of the absorption coefficient becomes significant compared with the scattering coefficient. Since $\gamma_s = 4\pi n''/\lambda$, where $n''$ is the imaginary part of refractive index, and $\lambda$ is the wavelength of the propagating microwave, the increase of $n''$ amplifies $\gamma_s$. Based on the formula of complex permittivity, $LWC$ is directly proportional to the imaginary part of permittivity, which is the main contributor to the increase of $\gamma_s$. Therefore, $r$ and $t$ tend to zero because $r_0$, the reflectivity of a layer of infinite thickness and $t_0$, the one way transmissivity, asymptotically tend to zero.

Now, let us consider snow density and grain size effect on $r$, $t$, and $e$. First, snow density and grain size both influence the scattering coefficient $\gamma_s$ based on [22, eqs. (59) and (60)]. However, the grain size is 1 mm for this experiment and is only valid with [22, eq. (59)]. If the grain size increases, $\gamma_s$ increases, resulting in a decrease of $t$ and an increase of $r$. On the other hand, with increasing snow density caused by the introduction of $LWC$, the absorption coefficient, $\gamma_t$ increases, which is an additional effect on $D_1$. Therefore, the effects of snow grain size and density are nonlinear compared with $LWC$ on microwave radiative transfer within a snowpack.

In summary, with the asymptotic increase of $e$, the two remaining parameters $r$ and $t$ become negligible. This implies that the snowpack behaves like an opaque medium as the $LWC$ reaches a certain level in the microwave spectrum that explains why $D_n$ converges to a specific $Tb$ as a function of the $LWC$ inside the snowpack.

### E. Why Does $(1 - s_n)$ Monotonically Decrease?

The second part of the question formulated in Section II-C can be rephrased as: “Why does $s_n$ increase with $LWC$?” The equation of $s_n$ using Snell’s law is

$$s_1 = \frac{\cos(\theta_2) - n_2/n_1 \cos(\theta_1)}{\cos(\theta_2) + n_2/n_1 \cos(\theta_1)}$$

where $s_1$ is an interface reflectivity between the top snow layer and the atmosphere, $\theta_2$ is the incidence angle between the radiometer antenna and a flat earth, $\theta_1$ is the angle between the atmosphere and the snowpack, and $n_2/n_1$ is the ratio of the complex refractive index between air and snow. Here, the index of snow layers follows a top-down approach so that index 1 is the atmospheric layer while index 2 denotes the snow layer. Even though MEMLS uses the permittivity form to determine the reflection coefficient, here the refractive index instead of the real part of the permittivity by noting that $n = \sqrt{\varepsilon}$. Fig. 3(c) shows the ratio of the real part of the refractive index between the snowpack and the atmosphere indexed 1 and 2, respectively in (10). As the $LWC$ increases, the ratio $n_2/n_1 = n_{\text{air}}/n_{\text{snow}}$ decreases since $n_2$ becomes significantly large due to the $LWC$ in the snowpack. As the snowpack becomes increasingly wetter, its refractive index increases while the refractive index of the atmosphere is kept constant at 1.0. Thus, the Fresnel coefficient for the horizontal polarization increases. The increasing reflectivity in Fig. 3(c) explains the decrease of the reciprocal of the refractive index of the snowpack along with the increase of snow density. Based on this chain of events in radiative properties triggered by changes in the $LWC$, the increasing trend in the interface reflectivity can be traced back and attributed to the increase of $LWC$ within the snowpack. To support the increasing interface reflectivity, Fig. 3(d) shows the amplifying trend of $s_n$ with increasing $LWC$. These results are consistent.
III. Coupled Model Application to CLPX 2002–2003

References [39] and [40] previously evaluated a coupled model using single and multi-layer snowpack formulations to simulate the Tb in different hydrologic regimes. Snow hydrology subroutines are partially from an energy balance module from [41] and also from [10] for multi-layer and mass balance modules. Detailed algorithms and applications of the coupled hydrology-microwave emission model are provided by [39] and [40]. The mass balance equation of each snow layer is expressed by [39], [40]

\[
\frac{dh_{\text{swe}}^n}{d\tau} = \frac{1}{\rho_w} (P_{\text{sn}} + xP_{\text{r}}) - \Phi_{\text{sn}}^j, \quad j = n, \text{ at top layer} \quad (7)
\]

\[
\frac{d\text{SWE}^j}{d\tau} = \Phi_{\text{sn}}^{j+1} - \Phi_{\text{sn}}^j, \quad j = 1 \text{ to } n - 1 \quad (8)
\]

where \(\rho_w\) [kg m\(^{-3}\)] is the density of water, \(P_{\text{sn}}\) [kg (m\(^2\))\(^{-1}\)] is precipitation in the form of snow, \(\text{SWE}^j\) [m] is the snow water equivalent \((\text{SWE})\) at layer \(j\), \(\tau\) [s] is the time step, and \(xP_{\text{r}}\) [kg (m\(^2\))\(^{-1}\)] is the accumulated precipitation from rain-on-snow events. \(P_{\text{sn}}^j\) and \(xP_{\text{r}}^j\) are zero when \(j\) is not equal to \(n\), and \(\Phi_{\text{sn}}^j\) [m s\(^{-1}\)] is the snowmelt flux from layer \(j\) to \(j - 1\)

\[
\Phi_{\text{sn}}^j = \rho_w \frac{dh_{\text{swe}}^j}{d\tau} \quad j = 1 \text{ to } n \quad (9)
\]

where \(h_{\text{swe}}^j\) [m] is the depth of liquid water from snowmelt or rainfall in layer \(j\). This is obtained from the energy balance equation. The maximum retention capacity of liquid water in each layer is a model parameter typically specified as 5%. Above this value the liquid water is released to the layer underneath. Intermittent melting is only triggered in the top layer from diurnal energy transfers such as solar radiation, latent and sensible heat, and energy transfer from the adjacent layers. \(h_{\text{swe}}^j\) (\(h_{\text{swe}}^j = (\rho_{\text{w}}/\rho_{\text{sn}})\text{SWE}^j\)) can be calculated by integrating across the \(n\) snowpack layers. Layer properties such as a volumetric liquid water content \(\text{LWC}^j\), ice water content \(i_{\text{w}}\), snow depth \(h_{\text{sd}}\) [m], \(\text{SWE}\) [m], and snow density \(\rho_{\text{s}}\) [kg m\(^{-3}\)] are updated as the simulation marches forward.

To evaluate the sensitivity of \(Tb\) to \(\text{LWC}\) in realistic conditions, the coupled snow hydrology-microwave emission model is applied to CLPX 2002–2003. A detailed description of CLPX is provided by the National Snow and Ice Data Center (NSIDC) CLPX website (http://nsidc.org/data/clpx/ and [31]). Interdisciplinary research on cold regions hydrology from remote sensing, numerical modeling and field measurements was conducted between the fall of 2002 and the 2003 spring season over the complex terrain of the Colorado Rocky Mountains, United States. The application here is limited to two CLPX 2002–2003 Meso-cell Study Areas (MSAs): the Fraser and the North Park MSAs. The Fraser MSA is located at 39.89° N and 105.87° W at 3597 m above sea level (a.s.l). The North Park MSA is located at 40.68° N and 106.28° W at 2435 m a.s.l. The Fraser MSA is in forested, complex terrain whereas the North Park MSA resides in a grassland plateau (Fig. 4).
The terrain at the Fraser MSA is also four times steeper than at the North Park MSA. The two application sites represent very different topographies and land cover conditions in the Colorado Rockies, but still enabling us to test the transferability and robustness of the findings from the idealized single-layer simulation to real-world cases with more than a 1000 m difference in elevation. Due to elevation difference, the Fraser MSA is much cooler than the North Park MSA. The Fraser MSA is covered by a variety of vegetation from grasslands to conifers while the North Park MSA is dominantly occupied by dry grasslands [31]. Details on the climatologies for the two CLPX MSAs are provided by [39]. Briefly, they report 10-m mean air temperatures of $-6.4\, ^\circ{}C$ and $-4.2\, ^\circ{}C$ from January to March 2003 at Fool Creek in the Fraser MSA and at Michigan River in the North Park MSA, respectively. In addition, average snow depths observed are 1.07 m and 0.01 m for the Fraser and North Park MSAs, respectively. The CLPX area is seasonally covered with snow during winter [42]; however, the North Park MSA experiences a “shallow and transient snowpack” whereas the Fraser MSA is covered by a “moderate snowpack.” Liston et al. [43] also summarize the winter climate and landscapes of two sites: the Fraser MSA has the highest elevation gradient along north to south. Southern mountains in the Fraser MSA are mostly rugged, while the northern part is mostly gentle with moderate slope changes. Northern parts of the mountains are predominantly alpine tundra or rocks, but the southern parts are dominated by conifers. Compared with the Fraser MSA, the North Park MSA is relatively flat and covered by sagebrush shrubland with meadows near rivers or streams.

The model forcing files are extracted from two sources: 1) meteorological tower data (air and soil temperature, precipitation, relative humidity, etc.) at the Local Scale Observation Station (LSOS) in the Fraser MSA where the GBMR-7 was located (http://nsidc.org/data/docs/daac/nsidc0165_clpx_gbm/index.html, [32], and [44]), and 2) RUC-40 meteorological forecast data (http://nsidc.org/data/docs/daac/nsidc0211_clpx_ruc40/, and [45]) for the Fraser and the North Park MSAs, respectively. There were several advantages in using these observational data for the model forcing, including the high sampling frequency (10 min) and the availability of soil temperature data below the snowpack. The GBMR-7 forcing files are available only up to March 2003. On the other hand, the RUC-40 data are available hourly and cover the entire snow season from October 2002 to June 2003. The following sections evaluate $Tb$ responses to changes in snow hydrology through coupled model simulations in both the Fraser and North Park MSAs, respectively. A single-layer snowpack is used in the Fraser MSA, whereas a multi-layer scheme is applied to the North Park MSA. For the balanced application of single and multi-layer simulations at the Fraser MSA, a multi-layer scheme is applied to the Fraser LSOS to compare simulated $Tb$s from MEMLS with the observed ones from GBMR. In addition, the maximum retention capacity of liquid water was imposed to be 5% of the $SWE$ [m] both for the Fraser MSA and GBMR-7 site, and 10% at the North Park MSA based on previous work [39] and [40].

Fig. 5 shows the $SWE$ and the $LWC$ simulated by the coupled model forced by the GBMR observations in the Fraser MSA. For the analysis of the “energy saturation” phase, the month of February 2003 is selected when the $LWC$ remains briefly at 0.01 m$^3$ m$^{-3}$ (solid rectangle). During early February 2003, $LWC$ reaches values below 0.01 m$^3$ m$^{-3}$, but the snowpack does not experience deep melting. This amount of $LWC$ corresponds to the early increase in $Tb$ previously discussed in the context of Fig. 2(b) and Section II-C.

When the $LWC$ increases, the simulated $Tb$ increases sharply [Fig. 6(a)]. Here, the results are for the Fraser MSA, but the model is driven by GBMR-7 meteorological data and not RUC40 reanalysis data. Concurrent SSM/I observations at 37.5 GHz h-pol. show similar behavior, though the amplitude of the coupled model simulated $Tb$ response is more sensitive to $LWC$ changes, particularly when $LWC \leq 0.005$ m$^3$ m$^{-3}$. Note, however, the difference in spatial resolution between the model simulations and the SSM/I pixel size as discussed by [39] and [40]. Because $D_n$ contributes to the $Tb$ in the “energy saturation” phase ($LWC \leq 0.005$ m$^3$ m$^{-3}$), it also shows the amplification of $Tb$ with response to the $LWC$. As discussed in Section II-D, increases in $e$ lead to increases of $D_n$ and $Tb$. Based on (2), $t$ and $r$ become negligible while $e$ converges asymptotically to unity when $LWC$ is well above the threshold [Fig. 6(b)]. The snow temperature remains constant at the melting point as the $LWC$ increases in the snowpack (not shown).

Another simulation example is the monotonic decrease of $Tb$ with higher $LWC$ in the “energy dampening” phase. In Fig. 5, the dotted rectangle indicates the period of six days from 14 to 19 March 2003 corresponding to this regime. Fig. 6(c) focuses on the simulated $Tb$ as well as the $LWC$ increases at the Fraser MSA in that period. In this case, the SSM/I observations also exhibit a monotonic decrease as $LWC$ increases. The arrow indicates the monotonic decrease of observed $Tb$ from SSM/I data when $LWC$ increases from 17 to 18 March. Specifically, after 16 March 2003 the SSM/I shows a distinct decreasing trend in observed $Tb$s. After the simulated $Tb$ increases after the 19 March due to an initial increase of $LWC$ and an increase in snow physical temperature, simulated $Tb$ starts to attenuate.
after the 19th of March when LWC surpasses 0.01 m$^3$ m$^{-3}$. This attenuation corresponds to energy dampening phase. Compared to $T_b$ simulation, SSM/I observations accurately captured the decrease of $T_b$ after the LWC exceeds 0.01 m$^3$ m$^{-3}$. As the LWC increases, $(1-s_n)$ decreases [Fig. 6(d)]. Consequently, $T_b$ decreases despite increases in $D_n$. As discussed in Section II-E, $s_n$ continuously increases even when the LWC increase beyond the onset of melt. However, when LWC > 0.005 m$^3$ m$^{-3}$, the decreasing trend in $(1-s_n)$ becomes significant and results in the monotonic attenuation of snowpack $T_b$.

B. North Park MSA

For the North Park MSA, the RUC-40 forecast-assimilated data are used to force the coupled model. It is advantageous to use the RUC-40 because the simulation can cover the entire snow season including the periods of snow accumulation, peak and melt. Compared with the Fraser MSA, the SWE at the North Park MSA reaches only 0.2 m during the winter of 2002/2003 given the difference in altitudes between the two sites (Fig. 7). Two distinct periods are selected to investigate the $T_b$ responses to changes in LWC. As much as the LWC exhibits a diurnal cycle, the simulations and observations reveal peaks in $T_b$ in response to a concurrent diurnal cycle in emissivity $e$ [Fig. 8(a) and (b); also note this period is shown with a solid rectangle in Fig. 7 at the North Park MSA]. When the LWC < 0.01 m$^3$ m$^{-3}$, which falls under “energy saturation” phase, the $T_b$ increases. Both simulated and observed $T_b$s are amplified in early March. After 9 March, the LWC starts to increase significantly, and the simulated and observed $T_b$s also amplify. Then, $e$ initially increases with LWC but also abruptly decreases when LWC exceeds 0.01 m$^3$ m$^{-3}$ [Fig. 8(b)]. An analysis of $e$ behavior shows clearly that when the LWC changes, $e$ leads the $T_b$ response. However, when the LWC increases above 0.01 m$^3$ m$^{-3}$, a monotonic decrease of $T_b$ occurs in both the simulations and observations.

In the “energy dampening” phase, $T_b$ initially increases on 28 May 2003 [Fig. 8(c)]. This period corresponds to the dotted rectangle in Fig. 7. However, subsequent $T_b$ values show a decreasing trend for both the simulations and observations even when the diurnal maxima of LWC exceed 0.01 m$^3$ m$^{-3}$. The attenuation of $T_b$s is also explained by the concurrent increase in $s_n$ [Fig. 8(d)].

In addition to the snowpack scale analysis, the multi-layer snow hydrology model allows us to investigate both the “energy saturation” and “energy dampening” phases within the various layers. In other words, it is possible to identify which layer contributes most to the change of radiometric responses when the snowpack changes in mass and heat content. Here, the results presented are limited to two days from 11 to 12 March 2003. The maximum liquid water retention is set at 5%. The LWC is more easily released from the snowpack than earlier in the season. Note that Fig. 9 is based on simulations for...
Fig. 8. (a) Early increase of $T_b$ responses to the $LWC$ change for 12 days in March 2003 with black dots indicating SSM/I observations. (b) Emissivity at 37.5 GHz h-pol. with response to the $LWC$. (c) $T_b$ response to the $LWC$ change during five days in May 2003 with black dots indicating the SSM/I observations. Note that the arrow indicates the attenuation of $T_b$, and (d) interface transmissivity, $(1 - s_n)$, response to the $LWC$ change during five days in May 2003. Note that all results are for the North Park MSA.

Fig. 9. (a) Emissivities of top, top-1, and top-2 layers with response to the $LWC$. (b) Layer emissivities with bulk emissivity of the snowpack. (c) Real parts of permittivity of top, top-1, and top-2 layers with response to the $LWC$, (d) Real parts of permittivity and simulated $T_b$ at 37.5 GHz h-pol. Here, black dots indicate SSM/I observations and attenuated SSM/I $T_b$ is within dotted circles. Note that all results are for the North Park MSA.

As the $LWC$ increases, the top layer initially responds followed by an increase in the emissivity of the uppermost layer [Fig. 9(a)]. When the top layer $\epsilon_n$ peaks, the simulated $T_b$ increases [Fig. 9(b)], and the $LWC$ remains at or below 0.01 m$^3$ m$^{-3}$, that is in the “energy saturation” phase. Immediately thereafter, there is a strong decrease in $T_b$ as $LWC$ increases. An analysis of the layer by layer evolution of permittivity in Fig. 9(a) shows that the second uppermost layer $(n - 1)$ is more sensitive to change of $T_b$ and $LWC$ as compared to the top $(n)$ and bottom layer underneath $(n - 2)$.

To evaluate the multi-layer process in the “energy dampening” phase, other dates are selected for analysis. This is because the number of snow layers was only one for the “energy dampening” phase examined in Fig. 8(c) and (d). Let us now focus on the period 1 to 5 May 2003 with three snow layers and $LWC > 0.01$ m$^3$ m$^{-3}$ [Fig. 9(c)]. This period is corresponded
to the arrow in Fig. 7. Note that a snow layer is divided into two when the depth of the original layer exceeds 0.1 m of SWE in this case (see [40] for details). As discussed in Section II-E, the large, real part of the permittivity induced by the LWC is the main contributor to attenuating \( T_b \). The third layer has high LWC compared to the upper two layers as snowmelt percolates down in the snowpack. As the LWC increases, the real part of the permittivity increases for all layers. At 00:00 UTC on 2 May 2003 and near 12:00 UTC on 3 May 2003, the \( T_b \) for both the simulations and the SSM/I observations become significantly attenuated [Fig. 9(d)]. Concurrently, the real parts of the permittivity of the snow layers contribute to the increased attenuation of the snowpack at 37.5 GHz h-pol.

However, on 5 May 2003 when the LWC falls to zero, the real part of the permittivity for the second layer is constant while the top and third layer permittivity decrease [Fig. 9(c)]. This “sandwich” organization of permittivity can be evaluated with the distribution of peak microwave frequencies for radiometers at \( f = 19 \) GHz, 21 GHz, and 37 GHz. Note also the relatively constant snowpack properties of the wrapped “isolated” second layer.

C. GBMR-7 Radiometer

The GBMR was installed on a moving truck equipped with a radiometer antenna and therefore more than one observation could be obtained at different times of the day at the deployment site [32]. To investigate the diurnal change of \( T_b \) responses, a movable radiometer receives the microwave emissions from the snowpack at the GBMR-7 location. The 19 February 2003 is selected to evaluate the effect of surficial snowmelt on \( T_b \). The box plot in Fig. 10(a) shows the variability of observations obtained at a given time as well as the change of measured \( T_b \) at 37.5 GHz h-pol at different times of the day. Because the GBMR-7 site separately measured \( T_b \) with the truck-mounted radiometer, here, \( T_b \) is differentiated with \( T_{obs} \) in the figure legend. At this particular incidence angle, the swath of radiometer antenna spans from 150 to 210 degrees of azimuth angle, which explains the wide range of \( T_b \) at each time. While the \( T_b \) remains on average around 200 K from 4:00 to 13:00 Local Time, a large jump can be seen after 16:00 Local Time. We hypothesize that this is indicative of higher LWC in the snowpack. To examine the physical processes leading to the \( T_b \) increase, the coupled model was applied to the GBMR-7 site using the same forcing data set as in Section IV-A, but using a multi-layer formulation in this case. In Fig. 10(b), the model simulation shows that the LWC initially only increases slightly after 13:00 Local Time followed by a strong increase that corresponds to the “energy saturation” phase. Next, when LWC continuously increases after a short-duration drop below 0.01 m\(^3\) m\(^{-3}\), the \( T_b \) consistently decreases corresponding to the “energy dampening” phase. To investigate which is the layer that contributes to the increase of \( T_b \), the simulated layer absorption coefficients are shown in Fig. 10(c). The top layer absorption coefficient is a main contributor to increase \( T_b \) by decreasing the extinction of microwave intensity from the snowpack. To explain the change of absorption coefficients, Fig. 10(d) shows the real part of the permittivity of each snowpack layer. As expected, the top layer permittivity initially increases with the corresponding \( T_b \) increase. However, the second and third layers from the top contribute significantly less to the peak of \( T_b \). However, the permittivities of those two layers slightly increase when the meltwater from the top layer percolates to the lower layers. The GBMR data [32] confirm the \( T_b \) increase due to an initial introduction of LWC in the snowpack, but the coupled model reveals entire processes of \( T_b \) increase and subsequent decrease from dry to wet (up to 0.01 m\(^3\) m\(^{-3}\)) snow regimes.
V. Conclusion

The modeling experiments and corresponding microwave satellite observations demonstrated the convex response of $Tb$ as a function of $LWC$ within the snowpack. Three separate case studies from Colorado, USA, also confirmed the initial increase and then the monotonic decrease of $Tb$ in response to $LWC$ using a coupled snow hydrology/microwave emission model. With increasing $LWC$ within the snowpack, the absorption coefficient plays a dominant role compared to the scattering coefficient. As the absorption coefficient abruptly increases with the appearance of $LWC$, $r$ and $t$ asymptotically become negligible. Subsequently, the layer emissivity, $e$ goes to unity, resulting in the upward brightness temperature within the snowpack to saturate. However, $s_n$ between the atmosphere and snowpack also increases since the real part of the permittivity of the snowpack amplifies. Since $Tb = D_e \times (1 - s_n)$, $Tb$ begins to decrease after the local peak at the initial point. This convex response is entirely caused by the $LWC$ within the snowpack.

As discussed by [27], the early peak of $Tbs$ for low $LWC$ can be used to detect the onset of snowmelt. However, more applications are possible by exploring the “energy dampening” regime above a $LWC$ threshold. As in [28], a significant amount of snowmelt or rain on snow can be detected by the decrease of $Tb$ from radiometer observations after the initial detection in the “energy saturation” phase. Then, the preventive evaluation of weak snow layers caused by initial snowmelt can be evaluated with the modeling approach. Here, scale issues of satellite-borne passive microwave radiometer from SSM/I must be addressed. Its footprint is of $1^\circ$ while the snow model and physics take place at the point scale. This discrepancy needs to be considered when comparing the model results with space-borne radiometer observations. To address limitations caused by scale differences, the GBMR-7 truck-loaded radiometer provides an alternative instrument to test current and future coupled models between microwave forward/backward and snow physics models.

Even with successful validations of the $Tb$ responses induced by the $LWC$ within the snowpack, there are other complex snowpack physics that need to be addressed for operational microwave remote sensing: 1) grain size evolution, and 2) porosity or density changes. To parameterize the grain size into the snow module in a coupled snow hydrology-microwave emission model, the equation for the thermodynamic mass and energy balance of the snow grains needs to be developed. The dry snow crystal metamorphosis has been well addressed, but wet snow growth processes have not been dealt with in most snow hydrology models. Daanen and Nieber [46] modeled numerically the liquid water flow inside the snowpack. The main mechanism of this flow is governed by Darcy’s law but numerical the liquid water flow inside the snowpack. When the liquid water flow accelerates downward in the snowpack, the snow grain size becomes sintered, resulting in larger particles. After addressing the proper simulation of the snow grain size, the scattering coefficient of the snowpack can be estimated. Because the absorption coefficient is an analytical function of the imaginary part of the refractive index of snowpack and wavelength propagated, the total extinction coefficient can be more precisely predicted from the estimation of the scattering coefficient. In addition, improved representation of snow grain size evolution can be used to improve the estimation of snowpack porosity and density, including the representation of the distribution of $LWC$ inside the porous snowpack with implications for wet snow metamorphism.

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