An Analytical Theory of Project Investment
A Comparison with Real Option Theory

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Abstract

Real option theory offers great insights about project investment. However, at present, most articles on this subject either use stylized numerical examples or adopt a purely conceptual approach to describe how option pricing can be used in capital budgeting. In this work, we observe a fundamental difference between the problems of project investment and financial option; mathematically speaking, project investment is a forward problem while option pricing is a backward problem. Starting from this simple observation, we develop an analytical theory of project investment, which provides an analytic formula that explicitly represents the relation among fixed costs, variable costs, uncertainty of the environment and the duration of a project, which is the core concern in most business projects and other economic decisions. This analytical theory provides more precise understanding of investment problems in a dynamic environment than real option theory.

Key Words: Analytical theory, project investment, real option

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1. Introduction

Real option theory captures some of the basic conditions in project investment, "namely irreversibility, ongoing uncertainty and some leeway in timing" (Dixit and Pindyck, 1994, p.23). "It seems clear that the incorporation of contingent claims analysis into capital budgeting decision-making promises to revolutionize the way corporations organize and assess their investment programs" (Megginson, 1997, p. 292). However, "at present, most articles on this subject either use stylized numerical examples or adopt a purely conceptual approach to describing how option pricing can be used in capital budgeting" (Megginson, 1997, p. 292). What, then, is missing in making the promise of a revolution into a reality?

In this paper, we will develop an analytical theory of project investment by observing a fundamental difference between project investment and the option problem. The problem of option pricing is to estimate option price when the payoff structure at the end of a contract is given. The problem of project investment is to estimate variable cost in production when irreversible fixed cost is invested or committed at the beginning of a project. Therefore project investment is an initial value problem while option pricing is an end value problem. Starting from this simple observation, we derive a formula of variable cost as an analytical function of fixed cost and other parameters. This is the main difference between this analytical theory and real option theory, where stylized numerical examples of possible combinations of fixed costs and variable costs of a project are used in discussing the returns of different project designs. This analytical theory provides more precise understanding of investment problems in a dynamic environment than real option theory. The basic ideas are as follows:

The evolution of the value of an economic commodity is often represented by lognormal processes in the literature. A business project that is designed to produce such a commodity involves fixed and variable cost. Irreversible fixed cost is spent or committed at the beginning of a project. Variable costs, on the other hand, are influenced by the state of output. For example, if consumer taste about a product changes very fast, variable costs in production are likely to be high. Since variable cost is a function of the output of a project, from the Feynman-Kac Formula, it satisfies a partial differential equation very similar to Black-Scholes equation (Øksendal, 1998). The difference is that this equation is a forward equation while the Black-Scholes equation is a backward equation.

To solve the equation for variable cost, we need to determine a proper initial condition, which we obtain through a thought experiment. With this initial condition, we solve the equation to obtain a formula of variable cost as an analytical function of fixed cost, uncertainty of the environment and the duration of a project. This analytical formula enables us to make a systematic comparison of returns of different types of investment under different kinds of market conditions, which refines the insights from the real option theory in many ways.

A major insight from real option theory is that companies have options to wait on projects. The higher the uncertainty, the higher is the value of waiting. This analytical theory offers a more precise understanding of this option value. From this theory, it can be derived that as uncertainty decreases, variable costs decrease substantially among high fixed cost projects and change little for projects require low fixed costs. Therefore, the option to wait is not only related to market uncertainty, but also to the required level of fixed cost of projects. If the fixed costs of projects are low, small companies will enter the fields early. Thus the option value of waiting mainly concentrates on projects with high fixed costs. For example, projects in natural resource exploration generally require high initial capital investment because of the natural environment, and projects in the pharmaceutical industry require high cost in research and development because of the regulatory environment. This explains why it is in the industries with high entry barriers that the real option theory is often applied.

Real option theory states that options to wait arise from the earlier investment made by companies (Dixit and Pindyck, 1994). Know-how, brand recognition and other company specific resources often allowed established companies to delay irreversible investment until the expected rate of return over a project is much higher than the cost of capital. On the other hand, despite the huge financial resources of large firms, they often are unable to compete effectively against small and new companies in emerging industries with great profit potential. For example, the champions of the IT revolution, such as Microsoft, Intel, CISCO, Oracle, and AOL, are all relatively new companies that reaped tremendous profits by being able to quickly respond to and take advantage of new market opportunities. This analytical theory provides reconciliation of the apparent inconsistency between established companies' options to wait in some cases and their inability to compete effectively in other cases. By necessity, large firms adopt more rigorous check and balance in evaluating and developing projects than small firms, which often increase fixed cost of projects. Since higher fixed cost projects enjoy advantage in significantly lower variable cost in production only when the uncertainty is low, large and established companies enjoy options to wait on projects in their own fields and are less effective in entering new markets with high uncertainty. This theory offers a clear understanding that the option to wait in one field can be a detriment to entry in another field.

The paper is organized as follows. Section 2 derives the analytical results of project investment. Section 3 discusses the return of various projects under different environments and explains how this analytical theory can refine many results of real option theory. Section 4 provides an example to illustrate the advantage of this new theory and offers some discussion how this theory may be improved. Section 5 concludes.

2. An analytical theory of project investment

Suppose S represents economic value of a commodity, r, the expected rate of change of value and σ, the rate of uncertainty. Then the process of S can be represented by the lognormal process

$$\frac{dS}{S} = rdt + \sigma dz . \tag{1}$$

The production of the commodity involves fixed cost, *K*, and variable cost, *C*, which is a function of *S*, the value of the commodity. If the discount rate of a firm is *q*, from the Feymann-Kac formula, (Øksendal, 1998, p. 135) the variable cost, *C*, as a function of *S*, satisfies the following equation

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - qC \qquad (2)$$

with the initial condition

$$C(S,0) = f(S) \qquad (3)$$

To determine *f(S)*, we perform a thought experiment about a project with a duration that is infinitesimally small. When the duration of a project is sufficiently small, it has only enough time to produce one piece of product. In this situation, if the fixed cost is lower than the value of the product, the variable cost should be the difference between the value of the product and the fixed cost to avoid arbitrage opportunity. If the fixed cost is higher than the value of the product, there should be no extra variable cost needed for this product. Mathematically, the initial condition for the variable cost is the following:

$$C(S,0) = \max(S - K, 0) \qquad (4)$$

where *S* is the value of the commodity and *K* is the fixed cost of a project. When the duration of a project is *T*, solving equation (2) with the initial condition (4) yields the following solution

$$C = Se^{(r-q)T} N(d_1) - Ke^{-qT} N(d_2) \tag{5}$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a standardized normal random variable. When the discount rate of the firm is equal to *r*, the rate of change of the commodity value, formula (5) takes the same form as the well-known Black-Scholes (1973) formula for European call options

$$C = SN(d_1) - Ke^{-rT} N(d_2) \tag{6}$$

This result is easy to understand. When the discount rate, *q*, is equal to *r*, Equation (2) becomes

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{7}$$

while the Black-Scholes equation is

$$-\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{8}$$

The only difference between the two equations is the direction of time. Both the evolution of share prices and value of economic commodities are represented by lognormal processes. For a financial option, the strike price at the end of the contract is known. The problem in option theory is to estimate the option price when the strike price, as well as several other parameters, is given. For a business project, irreversible fixed investment is determined at the beginning of a project. The problem in project investment is to estimate variable costs when fixed costs, as well as other factors, are given. Mathematically speaking, the problem in option theory is to solve a backward equation derived from a lognormal process for option prices with a known end condition, the strike price; the problem in project investment is to solve a forward equation derived from a lognormal process for variable costs with a known initial condition, the fixed investment. The similarity between these two problems explains why the option theory becomes so important in understanding project investment and other economic problems.

It should be noted that the interpretations of uncertainty, σ, differ in an option model and project investment. In an option model, the uncertainty is purely about the underlying assets. In a project, uncertainty is about the whole production system, of which the uncertainty of the underlying asset is only a part. For example, suppose both Microsoft and a small software company plan to develop same type of application software based on the Windows operating system. The uncertainty of demand for this type of software affects both companies. At the same time, the small software company also faces uncertainty about the upgrading of the Windows operating system, which affects the developers in Microsoft less as they are better informed.

Formula (5) is the main result of the paper. It provides an analytical formula of variable cost as a function of fixed cost, uncertainty, duration of project and discount rate of a firm. A new theory is ultimately justified by its implications. We will look at the properties and implications of this theory. For simplicity, we will only examine formula (6), the special case when the discount rate is equal to r. Several properties can be derived from (6). First, when fixed costs, K, are higher, variable costs, C, are lower. Second, for the same amount of fixed cost, when the duration of a project, T, is longer, variable cost is higher. Third, when uncertainty, σ, increases, variable cost increases. Fourth, when fixed cost approaches zero, variable cost will approach to the value of the product. Fifth, when the value of a product approaches zero, variable cost will approach zero as well. All these properties are consistent with our intuitive understanding of production processes.

3. An analysis of returns of projects and firms

Unlike a conceptual framework, this analytical theory enables us to make precise calculation of returns of different projects under different kinds of environments. First, we examine the relation between fixed cost and variable cost at different levels of uncertainty. For example, a product can be manufactured with two different technologies. One needs ten million dollars of fixed cost and the other needs one hundred million fixed cost. Assume the other parameters are unit value of the product, to be one million, discount rate, to be 10% and duration of the project, to be twenty-five years. When uncertainty of the environment is 30% per year, variable cost for the low fixed cost project is 0.59 million and variable cost for the high fixed cost project is 0.14 million, calculated from (6). When uncertainty of the environment is 90% per year, variable cost for the low fixed cost project is 0.98 million and variable cost for the high fixed cost project is 0.94 million. In general, as fixed costs are increased, variable costs decrease rapidly in a low uncertainty environment and decreases slowly in a high uncertainty environment. This is illustrated in Figure 1.

Next we discuss the returns of investment on different projects with respect to the volume of output. K is the fixed cost of production and C is the variable cost. Suppose the volume of output during the project life is Q, which is bound by production capacity or market size. We assume the present value of the product to be S and variable cost to be C during the project life. Then the total present value of the product and the total cost of production are

$$SQ \text{ and } CQ + K \tag{9}$$

respectively. The return of this project can be represented by

$$\ln\left(\frac{SQ}{CQ + K}\right) \tag{10}$$

Continuing the example on two technologies with different fixed costs, we now discuss how the expected market sizes affect rates of return. Suppose the level of uncertainty is 30% per year and other parameters are the same. If the market size is 100, the return of the low fixed cost project, calculated from (10), is 37% and the return of the high fixed cost project is -12%. When the market size is 400, the return of the low fixed cost project is 48% and the return of the high fixed cost project is 97%. Figure 2 is the graphic representation of (10) for different levels of fixed costs. In general, higher fixed cost projects need higher output volume to breakeven. At the same time, higher fixed cost projects, which have lower variable costs in production, earn higher rates of return in large markets.

From the above discussion the level of fixed investment in a project depends on the expectation of the level of uncertainty of production technology and the size of the market. When the outlook is stable and market size is large, projects with high fixed investment earn higher rates of return. When the outlook is uncertain or market size is small, projects with low fixed cost breakeven easier.

Projects are undertaken by firms, which often utilize existing assets to help reduce costs in producing or marketing new products. For example, Microsoft often bundles its application software together with its Windows operating system. This effectively reduces the cost of marketing. In general, new products from large firms often enjoy the benefit of brand recognition, which reduces variable cost in marketing. At the same time, costs of projects are often affected by the characteristics of firms. In general, ownership and management are less integrated in large firm than in small firms. Therefore, large firms adopt more rigorous check and balance systems in corporate control than small firms. This added cost of monitoring often increases the cost of projects. Therefore higher fixed cost large firms generally concentrate on large and stable markets while lower fixed cost small firms thrive in uncertain niche markets. Firms of different sizes will choose different types of markets. For example, large banks, as high fixed cost systems, concentrate on standard financial products with high volumes, such as the credit card business. Small community banks, as low fixed cost systems, concentrate on small business loans that need individual judgment case by case. (DeYoung, Hunter and Udell, 2003)

Many firms, especially well-established firms, often require the rate of return on investment to be substantially higher than the cost of capital, that is, they will only consider projects with expected net present value (NPV) significantly higher than zero. Two questions arise from this empirical regularity. First, why are there plentiful high positive NPV projects in a competitive world? Second, why will firms not consider many projects with positive NPV?

From the real option theory, the existence of many high net present value projects in some companies comes from the option to wait, which is derived from the previous fixed investment (Dixit and Pindyck, 1994, p. 9). This analytical theory provides clear understanding about the nature of this option. From (6), firms with high fixed assets enjoy lower variable costs, which is the source of the option to wait. How long a firm can wait depends on the level of its variable cost, which is a function of fixed assets and uncertainty. Take Microsoft for example. From time to time, Microsoft develops its own application software to replace popular application software from other vendors, such as Word for WordPerfect, Excel for Lotus 123 and Internet Explorer for Netscape. Why can

Microsoft have this option to wait until other companies have developed popular products? It is because Microsoft has accumulated large amount of fixed asset in its dominant operating system. Application software is developed on top of operating systems, which are upgraded from time to time. Compared with outsider developers, software development inside Microsoft faces less uncertainty in upgrading application software to take advantage of the improvement from new versions of operating systems. Microsoft also bundles application software with its operating system in distribution, which makes its variable cost in distributing application software lower than other software makers who have to market their products separately. Because of this option to wait, Microsoft saves money on R&D and marketing research to develop software that has significant market potential. It can wait until other companies develop highly profitable application software before it decides to enter the market and internalize popular products into its Windows operating system. The extremely low variable costs of Micorsoft's application software, which are the results of existing assets, are the source of high NPV for many projects in Microsoft.

In the standard literature, it is often stated that firms should undertake a project when its NPV is positive. In reality, firms, especially well-established firms, restrict themselves to projects with significantly positive NPV. This analytical theory offers a very simple understanding of this pattern. From Figure 1, increase of uncertainty affects the performance of high fixed cost projects far more than low fixed cost projects. We will illustrate this with the same example of two projects with fixed costs of 10 million and 100 million respectively. Suppose the market size is 200 and other parameters are the same as before. If the level of uncertainty is 30% per year, the return of the low fixed cost project, calculated from (10), is 44% and the return of the high fixed cost project is 46%. When the level of uncertainty is 80% per year, the low fixed cost project breaks even and the high fixed cost project sustains a loss of 32%. Therefore high fixed cost projects are much more sensitive to uncertainty than low fixed cost projects. It is often difficult to isolate risk of a single project from risk of the whole firm (Stultz, 1999). That is why large firms, which often undertake high fixed cost projects, only consider projects with high expected NPV and low uncertainty.

While high fixed cost large firms are highly competitive and enjoy options to wait on projects in their own fields, they are less effective in entering new markets with high uncertainty. From Figure 1 and 2, high fixed cost systems need large market size to break even and are more sensitive to uncertainty. Small firms, however, can explore niche markets easily because their low fixed costs make them more flexible. This is why small firms account for a disproportionately high share of innovative activity (Acs and Audretsch, 1990). Empirical evidence shows that small firms are less sensitive to uncertainty than large firms (Bulan, 2003). Despite the significant financial resources of large firms, it is usually the small firms that pioneer in emerging industries.

From the above discussion, we find that different firms require different rates of return for their investment. Small firms requires lower rate of return for they are less sensitive to market changes. At the same time, they have less option to wait because higher profit potential will trigger the entry of more competitive large firms. Large firms require higher rate of return for their investment. They can wait longer than small firms but have less freedom to explore new territories, where uncertainty is high.

We make some further discussion on how this analytical theory differs from earlier works. Equation (2) is of first order in temporal dimension. This indicates that economic systems are intrinsically evolutionary. Currently many works on real options resort to steady state solutions in solving Black-Scholes equation. Others, such as Dixit and Pindyck (1994), avoid Black-Scholes equation altogether. Both methods settle into a general equilibrium framework. Our analysis shows that different systems have different kinds of competitiveness in different kinds of market environments. Even if steady states are reached, a change of market environment will disrupt the balance in old systems. Hence steady states are rarely attained and can not last long. This is consistent with casual observation and Schumpeter's theory of creative destruction. On a more fundamental level, this is consistent with the fact that all living systems are non-equilibrium systems. (Prigogine, 1980)

4. An example of application and further discussion

We will apply the theory to an example from Dixit and Pindyck (1994) to illustrate how the new theory can refine the insight from real option theory. Dixit and Pindyck provided an example on entry and exit in the copper industry. Some basic information about the example is the following:

> We will consider a facility that produces 10 million pounds of refined copper per year. To keep the analysis simple, we will ignore the fact that the mine's reserves are limited and will eventually run out; we will assume instead that the mine can operate forever. (This is not too extreme an assumption, since most copper mines can operate for at least 20 or 30 years.) A reasonable number of for the number for the cost of building such a mine, smelter, and refinery is I =$20 million, and for the cost of abandonment (largely for cleanup and environmental restoration) is E = $2 million. Average variable cost of production varies across firms in the United States, and even more so across different countries. We will set variable cost at C = $0.80 per pound, about the average for U.S. producers in 1992, but we will also vary this cost to determine its impact on the entry and exit thresholds. (For comparison, the average price of copper was about $1.00 in 1992, but over the 1985 – 1992 period it fell to as low as $0.60 per pound and rose to over $1.50 per pound.) (Dixit and Pindyck, 1994, p. 225)

Then they proceed to discuss the entry or exit decision when different levels of variable cost and volatility are given as independent variables. This is the standard methodology of "other things being equal". However, fixed cost, variable cost and volatility are not independent. A high quality mine will cost more to buy but cost less to operate, or using better equipment may reduce the variable cost in production. Variable cost is also a function of volatility. For example, if the demand of copper drops dramatically, workers have to be laid off, with high layoff costs and if demand of copper increases dramatically, recruiting and training new staff will be very costly. Furthermore, projects with different cost structure respond to different levels of volatility differently. From Figure 1, higher fixed cost projects are more sensitive to changes in volatility. In our theory, the relation among fixed cost, variable cost and volatility is captured explicitly by an analytical formula, which can be easily applied to different problems. When this is incorporated into Dixit and Pindyck's example, we can more precisely determine the timing of entry or exit because we model the impact of volatility on operating cost explicitly. Furthermore, management gains an option to choose different types of projects based on their judgment on future volatility. In fact, the timing of entry or exit is project specific.

While this analytical theory provides a simple and intuitive framework to understand investment problems, it is still at an early stage of development. Many of the assumptions can be relaxed to make the model more realistic. In particular, parameters in this model, such as fixed cost, uncertainty, discount rate and project duration are assumed to be independent while in reality these factors are interrelated. For example, systems often incur fixed costs to reduce uncertainty. In general, large companies often spend a greater amount of money on training to reduce uncertainty in the workplace than small companies. Warm blooded animals maintain a stable internal environment, which enables them to run faster or do other things more efficiently, at a cost of higher metabolic rates than cold blooded animals. At present this theory offers a qualitative reason why higher fixed cost systems, being more sensitive to uncertainty, will regulate their internal environment more vigorously than lower fixed cost systems. It also offers a quantitative solution to simple model problems. However, the ultimate purpose of an analytical theory is to give quantitative solutions to genuine empirical problems, which will only be possible after systematic work on data collection and analysis.

5. Concluding remarks

In this work, we develop an analytical theory of project investment by observing the fundamental difference between project investment and financial options. This analytical theory enables us to make precise calculation of returns of different projects under different kinds of environment. It extends and clarifies many ideas developed from real option theory. Since all production systems need fixed investment to lower variable costs, by providing an analytical theory about the relation among fixed costs, variable costs and uncertainty, this theory will help refine existing theories in many different fields.

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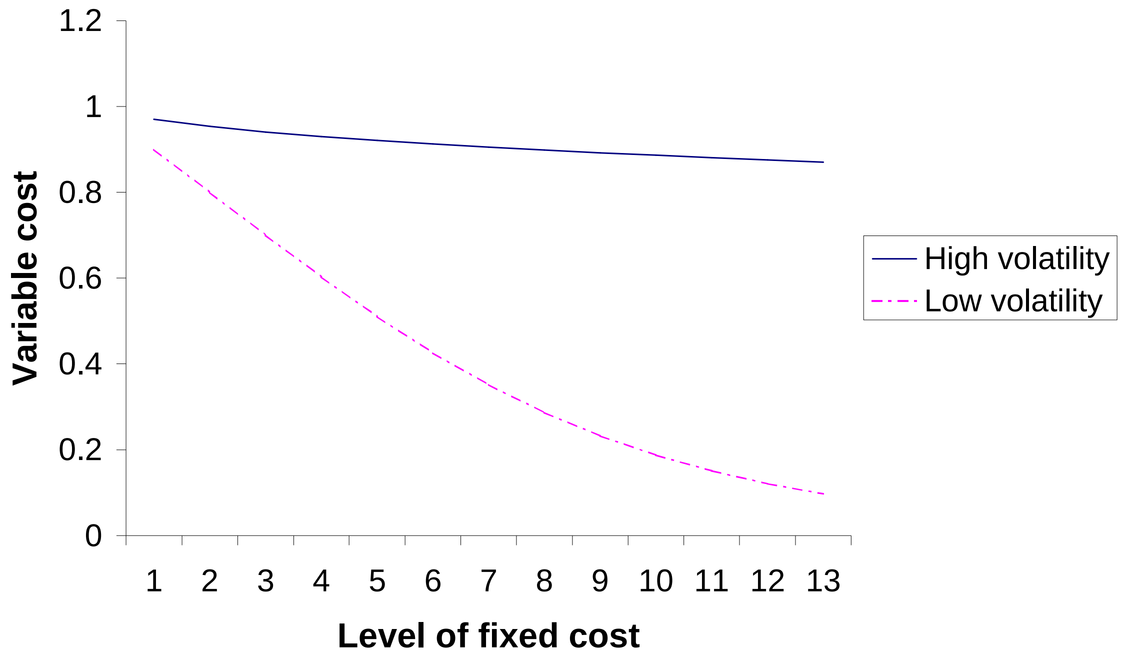
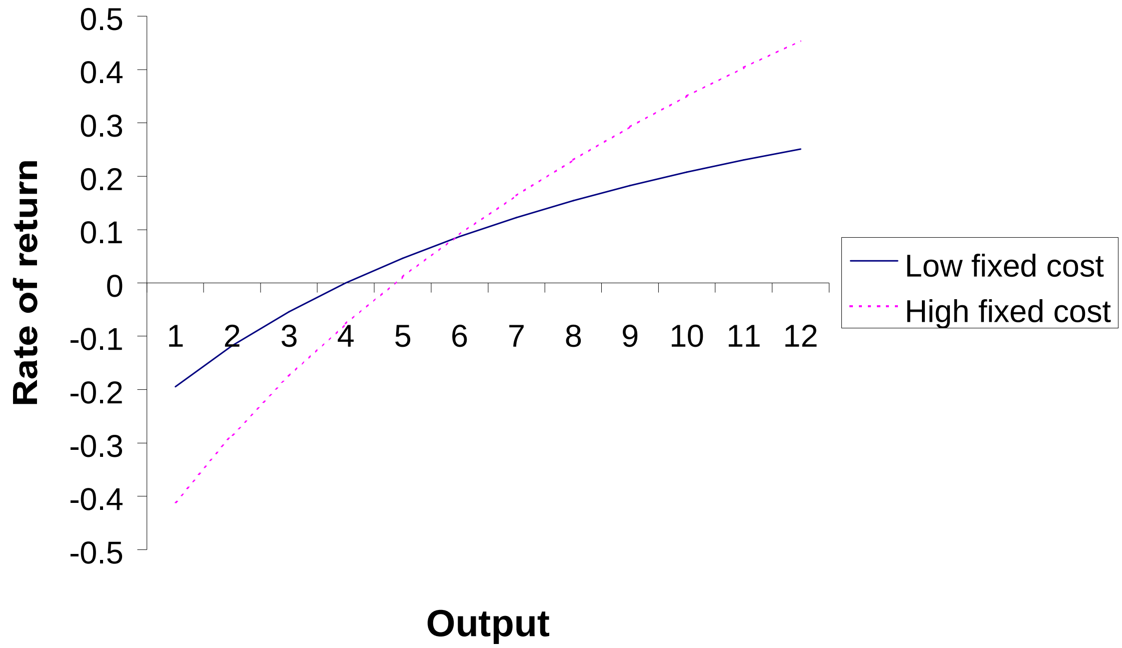
Figure captions

Figure 1. Level of uncertainty and variable cost: In a low uncertainty environment, variable cost drop sharply as fixed costs are increased. In a high uncertainty environment, variable costs change little with the level of fixed cost.

Figure 2. Output and return with different levels of fixed costs: For a large fixed cost investment, the breakeven market size is higher and the return curve is steeper. The opposite is true for a small fixed cost investment.


1.2
1
0.8
0.6
0.4
0.2
0
Variable cost
1 2 3 4 5 6 7 8 9 10 11 12 13
Level of fixed cost
High volatility
Low volatility


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Rate of return
1 2 3 4 5 6 7 8 9 10 11 12
Output
Low fixed cost
High fixed cost