Children as in-the-money options

An Application of Jing Chen's Thermodynamic Theory

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April 19, 2009

As we expand our experience into wilder and wilder regions of experience, every once in awhile we have these integrations in which everything is pulled together, a unification which in turn, turns out to be simpler than it looked before

Richard Feynman, 1981 BBC Interview

Introduction

In my first chemistry class I remember being struck by the second law of Thermodynamics, which basically states that entropy always increases. To sit back and think that we live in a world where there can never truly be a completely stable system; that there is always unused potential slipping away. Finance has been a field that seemed to embrace this same thinking. Rarely do we find ourselves in long periods of market stability and because of the potential for large profits and for large losses, continuous attempts have been made to better price market instruments. It seems that the fundamentals behind this strategy are to maintain a low entropy system. As we move beyond the field of finance we find similar missions, in fields such as biology, economics, psychology, however rarely do we find the same rigor in dealing with the physical constraint of entropy. It should be possible to apply models used in finance to tackle broader issues such as human overpopulation, education, and corporate strategy. The hopes of this paper are to first, introduce Jing Chen's Thermodynamic Theory that attacks multiple mainstream theories using real options and their analytical workhorse the Black-Scholes model, second, attempt to apply this thinking with a model that assesses the option value of having children, and finally conclude with a discussion of these results and implications.

Jing Chen'sTheory[1],[2],[3]

Chen's work revolves around the understanding that we are as free as the constraints set upon us by physical laws. The law central to the argument is that for systems to survive they must extract more low entropy resources from their surroundings than what is lost through continuous diffusion. Because of the explicit growth term and dissipation term, this function can be modeled with lognormal processes:

$$\frac{dS}{S} = rdt + \sigma dz \qquad (1)$$

Where S is the amount of negative entropy, or resources of an economic system, r is the rate of extracting the resources, and sigma is the rate of diffusion of negative entropy. For a system, be it an organism or a corporation, to have economic growth they must expand natural resources and reduce the diffusion of negative entropy. To do this, systems need to incur fixed, *K,* and variable, C, costs as a function of *S*, the value of the resource. Chen begins with the Feynman-Kac formula, where if the discount rate of a firm is *r*, the variable cost, *C*, as a function of *S*, is satisfied:

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \qquad (2)$$

Then Chen gets to the Black-Scholes formula from the initial condition:

$$C(S,0) = f(S) \qquad (3)$$

To determine *f(S)* it is assumed that a firm will only be able to produce one unit if production time is sufficiently limited. From there Chen can begin developing basic conditions. If the fixed cost is lower than the value of the resource, then to avoid arbitrage the variable cost should be the difference between the value of the resource and the fixed cost. However, if the fixed cost

[1] Adapted from Chen, Jing, "Economic and Biological Evolution: A Real Option Approach", June 2001

[2] Adapted from Chen, Jing, "An Analytical Theory of Project Investment A Comparison with Real Option Theory", June 2003

[3] Adapted from Chen, Jing, Galbraith James K, "A Biophysical Approach to Production Theory", working paper, February 2009

is greater than the value of the resource, then there should be no variable cost need for the resource. This brings us to a more familiar formula:

$$C(S,0) = \max(S - K, 0) \tag{4}$$

Where *S* is the value of a resource (negative entropy) and *K* is the fixed cost of a project. When we bring in time, *T*, solving equation (2), with condition (4), yields the Black-Scholes formula for European call options:

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{5}$$

where:

$$d_1 = \frac{\ln(S/K) + (r \ + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Chen now has an analytic formula of variable cost as a function of resource value, fixed cost, uncertainty, discount rate, and project duration; vital production variables he claims modern economics has shifted into the periphery for lack of an analytic model.[4] From here he determines the return of a project to be:

$$\ln\left(\frac{SQ}{CQ + K}\right) \tag{6}$$

Where *Q* is the volume of output during, *S* is the value of the resource (making *SQ* total present value of resource), *C* is the variable cost, and *K* is the fixed cost (making *CQ+K* the total cost of production). Then the net present value of project is:

$$QS - (QC + K) = Q(S - C) - K \tag{7}$$

[4] Chen, Jing, Galbraith James K, "A Biophysical Approach to Production Theory", working paper, February 2009

With this derivation Chen feels comfortable making "quantitative calculations of the return to different projects under different environments."[5] This conclusion reaffirms my belief that there is great potential in applying the Real Option Theory into broad areas of interest.

Children as Options

We have identified that to survive all systems must retain more low entropy sources than is diffused, and in doing so these systems incur fixed and variable costs. A system, such as a family, incurs these costs to maintain stability. One low entropy resource families have had throughout history is children. Many families, particularly in agricultural or impoverished regions, continue to take on the fixed development costs of harvesting this resource for the possibility of some stability and future economic benefit. The family's ability to (sometimes and with some accuracy) exercise when you want a child, the fixed development costs of having a child, and the future earning potential of a child makes it a viable candidate to use option theory in an attempt to quantify the actual resource value a child is to a dependent family. Although, I would never say that one could truly quantify the value of child considering the lifetime of intangibles, I do think, as Chen does, that a quantitative economic analysis of their value could help explain current fertility rates in countries, issues with overpopulation, and warning signs about the education system.

Assumptions

In this analysis we assume that a family, consisting of a mother and father, are the owners of an American call option where the underlying asset is a child with future earning potential. The family is planning on having the child sometime between now and five years. The child's future earnings, *S,* will be considered value to the family and dependent on nested education options. These future earnings will continue to be value to the family until retirement at age 65. The

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[5] Ibid.

fixed development costs of a child, *K*, will be the total of housing, food, transportation, clothing, healthcare, education, and miscellaneous. The family will not borrow money to support the child and will have its opportunity cost measured by current savings account interest rates. The child's major stages are production (0-5 years), development (16-17 years), the nested option of higher education (0-2 years), and participation in work force (43-49 years). The child has the option to drop out at the legal age of 16 (where it is assumed no advanced education will be attained in the future), complete high school at 17, attend a 2 year school, 4 year public school, or a 4 year private school. Financial aid, school loans, fellowships, scholarships etc. were not considered for covering school expenses.

Method

Five options were analyzed with the last three being nested options. First, the option of having a child within zero to five years with the child dropping out at age 16 and beginning work. Second, the option of having a child within zero to five years, with the child completing high school and then beginning work. Third, having a child between zero and five years, completing high school, then getting an associate degree within zero to two years. Fourth, having a child between zero and five years, completing high school, and getting a 4 year bachelor's degree at a public university within zero to two years. Fifth, having a child between zero and five years, completing high school, and getting a 4 year bachelor's degree at a private university within zero to two years. To value these options we need to determine the fixed costs of development *K*, the future incoming earnings of the child discounted to the present *S*, the risk free rate *r,* the volatility σ, and option length *t*.

For each of these options, four different family income levels were used to determine the child's development costs: poverty-level, low, middle, and high income[6] (Exhibit one). To determine child development costs for a family in poverty, the same ratio of development cost to income for low income families was used plus a factor of 5% to account for increasing

[6] United States Department of Agriculture, "Expenditures on Children by Families, 2007," Publication number 1528-2007

fraction of cost as income decreases. The average income for a family in poverty in 2004 was \$12,015[7], this figure was grown at the current wage increase of 2%[8] to a 2008 figure to compare with the USDA data in exhibit one. With this data we can value the fixed cost *K* of development for each income level. The costs are grown at an inflation rate of 3% and discounted using the current savings interest rate of 3% to the time, *t,* when the option is exercised (0-5 years).

The costs of advanced education were also considered and would be the fixed cost, *K*, of a nested education option where *t* is between zero and 2 years. In 2008, the average costs were \$2,402 per year for an Associate's degree, \$6,585 for a public bachelor's degree, and \$25,143 for a private bachelor's degree, growing at 4.7%, 6.4%, and 5.9% respectively[9]. These costs were grown until the child reached 18 with the earliest possible being the year 2027 (t = 0). At age 18 the child has zero to two years to decide if further schooling is a right fit. The degree costs continue to grow and are discounted back to the *t* of the nested option. See exhibit two for average advanced education costs.

To determine the present value of future earnings of a child, *S*, income information based on educational attainment was again found from the US Census Bureau report in 2004. Median incomes for whites were used for 'Not high school graduate' (\$12,184), 'Graduate, including GED' (\$20,431), 'Associate degree (\$28,817), and 'Bachelor's degree (\$37,782)[10]. These values were grown at 2% for high school drop out and 2.5% high school graduate, 3.0% for associate degree, and 3.7% for public bachelor's and 3.7% for private bachelor's[11]. For the advanced

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[7] U.S. Census Bureau, Population Division, "Income Stable, Poverty Up, Numbers of Americans With and Without Health Insurance Rise" , Census Bureau Reports, Created: March 15, 2004

[8] Bureau of Economic Analysis, "State Personal Income," Press Release March 2009, accessed April 11, 2009 http://www.bea.gov/newsreleases/regional/spi/sqpi_newsrelease.htm

[9] College Board, "2008 Trends in College Pricing," Trends in Higher Education Series, summary on website accessed April 11, 2009: http://www.collegeboard.com/student/pay/add-it-up/4494.html

[10] U.S. Census Bureau, "Educational attainment in the US, " accessed March 2009, http://www.census.gov/population/www/socdemo/education/cps2004.html

[11] Bureau of Economic Analysis, "State Personal Income," Press Release March 2009, accessed April 11, 2009 http://www.bea.gov/newsreleases/regional/spi/sqpi_newsrelease.htm

education options the earnings were not total, but instead incremental above high school earnings in order to not double count income value. See Exhibit three for average income earnings. The earnings were not grown into perpetuity, but instead ceased when the child reached the retirement age of 65. The income was then discounted back to the present value also at the current savings rate of 3%. Although, it is valid to assume that families in higher income brackets may have higher investment hurdles, for comparability 3% was used, thus we can assume the *S* values to be an upward bound for a wealthier family.

The riskiness of the future economic value of the assets was difficult to determine and would have to be the expected volatility of the future earnings. In this model I assumed an arbitrary 20% volatility and tested the sensitivity of the option value to volatility.

With all the variables defined and valued it was possible to determine the premium on having a child. This was done by calculating the net present value, or *S*-*K*, for when *t* was equal to zero, and using equation 5 from Black-Scholes for when *t* was equal to one through five. For the advanced education nested options, the value of the nested option was first calculated again by using Black-Sholes for when *t* was one or two years and *S*-*K* for when *t* equal to zero. To reduce the amount of numbers only the highest value produced from the nested option would then be added to the *S* of the initial child production option which was then valued using either Black-Scholes or *S*-*K* depending on *t.* In the end each income bracket would have five options with six values for each possible *t* (zero to five). Please see exhibit four for excerpts of this modeling process.

Results and Discussion

For each income bracket and each expected education level, children are deep in the money options that should be exercised immediately by dependent families. For the large fixed cost options like initial development there was a 5.4% to 6.3% loss in value if the family waited five years to exercise. The private bachelor’s degree also had a cost of 6.3% if families waited the two years to exercise. In figure one we can see the maximum option values for each income bracket for each education level. The maximums for each were all when *t* was equal to zero. This would appear to make sense if we think of the rising development and advanced education costs. If you decide to wait to have children the costs do not wait with you. As of 2008 incomes were growing in line with inflation[12] whereas tuitions were growing around 5.5-6%[13]. However, it should be reiterated that the cost of waiting was not dramatic with 5-6% hurdles to clear if the timing is not right for you and your spouse.

Summary of Option Values Based on Income and Education					
Class	**Drop out**	**GED**	**Assoc**	**BA-pub**	**BA-pri**
Poverty	425,302	887,050	1,276,068	1,877,033	1,654,257
Low	406,355	869,019	1,258,037	1,859,002	1,636,226
Middle	391,305	851,543	1,240,562	1,841,527	1,618,751
High	365,804	824,939	1,195,360	1,814,922	1,592,146

Figure 1

For each level of education the families in poverty would get the most value. If we think back to Chen's theory this result would be in line with his belief that families in poverty have acquired the least resource value, thus incurring the least fixed costs required for stabilization. This allows them to maximize the benefit from cheaper development costs. In other words, the option value of children to poorer families is high because the strike price is so low. Compare this to the wealthy families who have amassed negative entropy whilst incurring higher fixed costs for this stabilization. These higher fixed costs result in wealthier families having to put in more, pay a higher strike price, to ensure the same outcome with smaller return. This can be visualized with a graph depicting the rate of return as a function of output in exhibit five. If we expand this idea and look at other wealthier regions in the world there is little doubt as to one of the reasons fertility rates in these regions have dropped. Chen notes that as the living standard i.e. higher fixed costs of a nation increases it makes investments in children riskier, thus resulting in more resources being put towards each child as a way to ensure payoff[14]. Yet,

[12] Bureau of Economic Analysis, "State Personal Income," Press Release March 2009, accessed April 11, 2009 http://www.bea.gov/newsreleases/regional/spi/sqpi_newsrelease.htm

[13] College Board, "2008 Trends in College Pricing," Trends in Higher Education Series, summary on website accessed April 11, 2009: http://www.collegeboard.com/student/pay/add-it-up/4494.html

[14] Chen, Jing, "Economic and Biological Evolution: A Real Option Approach", June 2001

one could fear overpopulation from the poorer regions of the world with families continually exercising their in-the- money options.

This disparity between the poverty class and the high income class is amplified in the 'drop out' category where there is a 16.3% difference between the value the poor obtain and what the rich obtain. The 16.3% gap is coming from the much larger *K* values for high income families and the smaller future earnings stream. If wealthy families enrolled their children in private schools this effect would be even more dramatic. This effect is dampened in the advanced education options because of the identical nested cost structure and the large value of future income. However, since diversity programs, scholarships, and financial aid are not included in this model, one can imagine multiple instances where the large value differences continue into advanced degree programs.

For each income bracket the largest gain in value from education option came from completing high school instead of dropping out. Each class doubled their option value with high income families topping out at a 126% return from staying in school for one more year to earn their GED. The worst loss of value from education options came from enrolling in a private college instead of a public college. Each class bracket lost near 12% of value in this decision even though incomes where growing at a rate of 3% for public and 3.7% for private. A similar result would be seen if private high schools were included in the analysis. So why do families continue to spend $20,000-$40,000 per year for an option that appears to have negative value in comparison? Common arguments stress better teaching, smaller classrooms, safety, extracurricular activities, and branding. However, based on this analysis these intangibles are not worth the cost. The option with more economic value is a public school education.

Figure one clearly demonstrates the value of getting an advanced education. There is an average $1.45 million difference for each income class in the lifetime value of having a bachelor's degree from a public school versus not graduating from high school. However, what we do not have time to discuss is the affordability of such an education. With the rising tuition costs doubling the current wage increases, this option may be out of reach for many families. This out of reach opportunity is demonstrated by the historical results of social mobility in the

United States, as seen in exhibit six. 41% of the lowest fifth quintile of a 1969 income group in the United States still remained in this lowest class in 1994.

As expected with far back-ended cash flows these results are quite sensitive to the discount rate used. Figure two shows the new maximum option values of adjusting the current 3% savings account rate to the MBA standard 10%. The results tell a similar story as above, with the maximum payouts still occurring when *t* is zero, the poverty class getting the most value from the options, and the largest jump in value being from dropping out to getting a GED. One glaring difference is that the option with the most economic value is now the associate's degree. As previously mentioned, more realistic modeling would most likely have class specific discount rates, where for low income families a low rate like the 3% is used and for higher income families a higher rate is used. Also, as in real option theory, the discount rate for the fixed costs would most likely diverge from the discount rate used on future assets, especially as this rate increases. However, a 3% rate was assumed for *S* and *K* in order to get a good sense of the upper bound for each of these options values.

Summary of Option Values Based on Income and Education					
Class	**Drop out**	**GED**	**Assoc**	**BA-pub**	**BA-pri**
Poverty	36,156	77,780	109,021	92,849	77,780
Low	23,608	65,531	96,772	80,600	65,531
Middle	13,436	54,566	85,807	69,636	54,566
High	4,761	37,025	67,223	52,095	37,025

Figure 2

The model was not especially sensitive to large changes in volatility. In fact, when the original assumptions were changed from 20% to 80% volatility, option value was often not affected by more than 10%. What did occur was a significant leveling out between the values of an option that was immediately exercised versus one that was not exercised until the fifth year. For higher incomes especially, the 80% volatility made holding onto the option for a couple of years worth more than exercising immediately, even if only by 4-5%.

Conclusions

Children as options are not that easy to value. There are numerous variables that have been left out that could be pursued further. However, the simplified results presented here seemed to paint an interesting picture of the economic benefit of children to dependent families. Interpreting the results with Chen's Thermodynamic Theory, we were able to move beyond the discussion of monetary value and begin to formulate theories on overcrowding, fertility, and the education system. This theory may allow us to expand our interpretations into broader realms linked by the inescapable constraints of entropy. Where there is a system incurring the fixed and variable costs of maintaining low entropy sources there is the potential to quantify the struggle and gain better understanding.

However, there are great dangers in over simplification; in attempting to cram problems into unfitting solutions and theories. The process of validation will depend on the success of multiple users' implementations and insights. Is Chen's theory breakthrough or is it just a different way of interpreting results from the Real Option Theory and the Black-Scholes model? It may be the latter, but nevertheless, his derivations and interpretations from the root laws of physics have given me new ways to think about fields that seemed less connected. For these reasons it is important to be advocates of fields such as financial engineering that are continuously innovating in realms that are far from any frame of equilibrium. The costs of innovation are rarely the innovation itself, but instead an inability to realize its limitations. Hopefully there will be more attempts to quantify, question, and verify theories in the social sciences and hopefully there will be more feedback and discussion on theories such as Chen's.

Exhibit One: Expenditures on Children by Families (Used for *K* in root options)

Age	Total	Housing	Food	Transportation	Clothing	Healthcare	Education	Misc.
		Pre-tax income less than $45,800 (Average = $28,600)						
0-2	$7,830	$2,970	$1,070	$930	$340	$600	$1,220	$700
3-5	8,020	2,930	1,190	900	340	570	1,370	720
6-8	8,000	2,830	1,530	1,050	370	650	810	760
9-11	7,950	2,560	1,830	1,140	420	710	490	800
12-14	8,830	2,850	1,930	1,290	700	720	340	1,000
15-17	8,810	2,300	2,080	1,730	620	770	580	730
Total	$148,320	$49,320	$28,890	$21,120	$8,370	$12,060	$14,430	$14,130
		Pre-tax income $45,800 to $77,100 (Average = $61,000)						
0-2	$10,960	$4,010	$1,280	$1,390	$410	$780	$2,000	$1,090
3-5	11,280	3,980	1,470	1,360	400	750	2,210	1,110
6-8	11,130	3,880	1,880	1,510	440	850	1,420	1,150
9-11	10,930	3,600	2,210	1,600	480	920	930	1,190
12-14	11,690	3,900	2,230	1,740	820	930	680	1,390
15-17	12,030	3,350	2,480	2,200	730	980	1,170	1,120
Total	$204,060	$68,160	$34,650	$29,400	$9,840	$15,630	$25,230	$21,150
		Pre-tax income more than $77,100 (Average = $115,400)						
0-2	$16,290	$6,380	$1,690	$1,950	$530	$900	$3,020	$1,820
3-5	16,670	6,340	1,910	1,910	520	860	3,290	1,840
6-8	16,310	6,240	2,310	2,060	570	990	2,260	1,880
9-11	15,980	5,970	2,680	2,150	620	1,060	1,580	1,920
12-14	16,810	6,260	2,820	2,300	1,030	1,070	1,210	2,120
15-17	17,500	5,710	2,970	2,780	940	1,120	2,120	1,860
Total	$298,680	$110,700	$43,140	$39,450	$12,630	$18,000	$40,440	$34,320

Source: adapted from United States Department of Agriculture, "Expenditures on Children by Families, 2007," Publication number 1528-2007

Exhibit Two: Average Costs of Advanced Education (used for *K* in nested options)

	Year	2 year	4 year public	4 year private
	2008	2402	6585	25143
0	2009	2498	7006	26626
1	2010	2598	7455	28197
2	2011	2702	7932	29861
3	2012	2810	8440	31623
4	2013	2922	8980	33489
5	2014	3039	9554	35464
6	2015	3161	10166	37557
7	2016	3287	10817	39773
8	2017	3419	11509	42119
9	2018	3556	12245	44604
10	2019	3698	13029	47236
11	2020	3846	13863	50023
12	2021	4000	14750	52974
13	2022	4159	15694	56100
14	2023	4326	16699	59410
15	2024	4499	17767	62915
16	2025	4679	18904	66627
17	2026	4866	20114	70558
18	2027	5061	21402	74721
19	2028	5263	22771	79129
20	2029	5474	24229	83798
21	2030	5693	25779	88742
22	2031	5920	27429	93978

Source of 2008 figures and growth rate: College Board, "2008 Trends in College Pricing," Trends in Higher Education Series, summary on website accessed April 11, 2009: http://www.collegeboard.com/student/pay/add-it-up/4494.html

Exhibit Three: Earnings Dependent on Education Level (used for *S*, with incremental earnings used for nested *S*)

	Total Earnings						Incremental Earnings		
	Drop out	GED	Assoc	BA	Age			Assoc.	BA
2003	$12,184	$20,431	$28,817	$37,782			2003	$8,386	$17,351
2004	12428	20942	29682	39180			2004	$8,740	$18,238
2005	12676	21465	30572	40630			2005	$9,107	$19,164
2006	12930	22002	31489	42133			2006	$9,487	$20,131
2007	13188	22552	32434	43692			2007	$9,882	$21,140
2008	13452	23116	33407	45308		Child option	2008	$10,291	$22,193
2009	13721	23694	34409	46985	1		2009	$10,715	$23,291
2010	13996	24286	35441	48723	2		2010	$11,155	$24,437
2011	14275	24893	36505	50526	3		2011	$11,611	$25,633
2012	14561	25516	37600	52395	4		2012	$12,084	$26,880
2013	14852	26153	38728	54334	5		2013	$12,574	$28,181
2014	15149	26807	39889	56344	6		2014	$13,082	$29,537
2015	15452	27477	41086	58429	7		2015	$13,609	$30,952
2016	15761	28164	42319	60591	8		2016	$14,154	$32,427
2017	16077	28868	43588	62833	9		2017	$14,720	$33,964
2018	16398	29590	44896	65158	10		2018	$15,306	$35,568
2019	16726	30330	46243	67569	11		2019	$15,913	$37,239
2020	17061	31088	47630	70069	12		2020	$16,542	$38,980
2021	17402	31865	49059	72661	13		2021	$17,194	$40,796
2022	17750	32662	50531	75350	14		2022	$17,869	$42,688
2023	18105	33479	52047	78138	15		2023	$18,568	$44,659
2024	18467	34316	53608	81029	16	Drop out	2024	$19,293	$46,713
2025	18836	35173	55216	84027	17	GED	2025	$20,043	$48,853
2026	19213	36053	56873	87136	18		2026	$20,820	$51,083
2027	19597	36954	58579	90360	19	Associate	2027	$21,625	$53,406
2028	19989	37878	60336	93703	20		2028	$22,458	$55,825
2029	20389	38825	62146	97170	21	Bachelors	2029	$23,322	$58,345
2030	20797	39796	64011	100765	22		2030	$24,215	$60,970
2031	21213	40790	65931	104494	23		2031	$25,141	$63,703
2032	21637	41810	67909	108360	24		2032	$26,099	$66,550
2033	22070	42855	69946	112369	25		2033	$27,091	$69,514

Source for original 2004 figures: U.S. Census Bureau, Population Division, “Income Stable, Poverty Up, Numbers of Americans With and Without Health Insurance Rise” , Census Bureau Reports, Created: March 15, 2004

Source for growth rates: Bureau of Economic Analysis, “State Personal Income,” Press Release March 2009, accessed April 11, 2009 http://www.bea.gov/newsreleases/regional/spi/sqpi_newsrelease.htm

Exhibit Four: Model Excerpts from poverty income bracket

Option 1: Dropout at 16								
assumptions								
income growth		2%						
savings account rate		3%						
Base Salary		18836						
Inflation		3%						
S	**t**	**17**	**18**	**19**	**20**	**21**	**22**	**23**
	0	18836	19213	19597	19989	20389	20797	21213
	1		19213	19597	19989	20389	20797	21213
	2			19597	19989	20389	20797	21213
	3				19989	20389	20797	21213
	4					20389	20797	21213
	5						20797	21213
S	**Discount**	**0.605**	**0.587**	**0.570**	**0.554**	**0.538**	**0.522**	**0.507**
446060	0	11396	11286	11176	11068	10960	10854	10748
441591	1		11286	11176	11068	10960	10854	10748
437166	2			11176	11068	10960	10854	10748
432783	3				11068	10960	10854	10748
428443	4					10960	10854	10748
424145	5						10854	10748

Excerpt from model, future income generated by high school dropout. Income grown until age 65, then discounted to the present, for *S.*

K	**t**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
	0	-2065	-2065	-1403	-1403	-1403	-1401	-1401
	1		-2127	-2127	-1446	-1446	-1446	-1443
	2			-2191	-2191	-1489	-1489	-1489
	3				-2257	-2257	-1534	-1534
	4					-2324	-2324	-1580
	5						-2394	-2394
K	**Discount**	**1**	**0.971**	**0.943**	**0.915**	**0.888**	**0.863**	**0.837**
-20759	0	-2065	-2005	-1323	-1284	-1247	-1208	-1173
-21381	1		-2127	-2065	-1363	-1323	-1284	-1244
-22023	2			-2191	-2127	-1403	-1363	-1323
-22684	3				-2257	-2191	-1446	-1403
-23364	4					-2324	-2257	-1489
-24065	5						-2394	-2324

Excerpt from dropout model showing yearly fixed costs of development, where *K* is grown and then discount to *t*

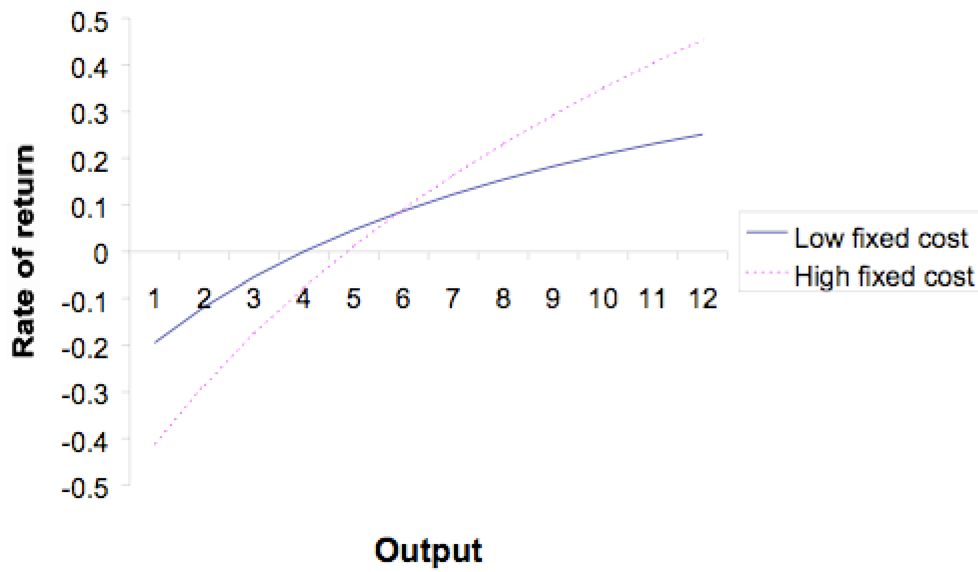
	No GED					
	S0	S1	S2	S3	S4	S5
S	446060	441591	437166	432783	428443	424145
X	20759	21381	22023	22684	23364	24065
rf	3%	3%	3%	3%	3%	3%
sigma	20%	20%	20%	20%	20%	20%
t	0	1	2	3	4	5
d1		15.39	10.92	8.94	7.77	6.98
d2	S-x	15.19	10.64	8.60	7.37	6.53
Opt val	$ 425,302	$ 420,842	$ 416,425	$ 412,052	$ 407,721	$ 403,432

Excerpt from dropout model calculating the value of the option at various *t*

	Bachelor's Public							HighSchool					
	S0	S1	S2					S0	S1	S2	S3	S4	S5
S	399140	392358	385692					908724	904313	899923	895555	891207	886881
X	10121	10425	10738		+ call value of Assoc		Smax	389019	389019	389019	389019	389019	389019
rf	3%	3%	3%				Stotal	1297743	1293332	1288942	1284573	1280226	1275900
sigma	20%	20%	20%				X	21675	22325	22995	23684	24395	25127
t	0	1	2				rf	3%	3%	3%	3%	3%	3%
d1	-	18.39	13.02				sigma	20%	20%	20%	20%	20%	20%
d2	S-X	18.19	12.73				t	0	1	2	3	4	5
Opt val	$ 389,019	$ 382,242	$ 375,580	max BA value added to prior			d1	-	20.55	14.59	11.96	10.40	9.34
				option			d2	S-X	20.35	14.31	11.61	10.00	8.89
							Opt val	$1,276,068	$1,271,667	$1,267,286	$1,262,927	$1,258,589	$1,254,273

Excerpt from dropout model showing nested option calculation. Where first the nested value of the bachelor's degree is calculated and then the maximum value of this option is added to the *S* of the root high school option.

Exhibit 5: Rate of Return as a Function of Output



Source: Chen, Jing, “Economic and Biological Evolution: A Real Option Approach”, June 2001

Exhibit 6: Income Mobility in the United States

Exhibit 9b Income Mobility in the United States—Controlling for Age

1969 income group	% in each quintile in 1994				
	Lowest	2nd	Middle	4th	Top
Lowest fifth	**41.0**	24.9	16.2	12.1	5.8
Second fifth	22.4	**24.7**	23.9	16.1	13.0
Middle fifth	16.9	21.0	**23.5**	22.8	15.9
Fourth fifth	11.3	18.5	19.7	**24.2**	26.3
Top fifth	9.5	10.6	16.6	24.5	**38.8**

Source: Adapted from Mishel, Bernstein, and Schmitt, *The State of Working America 2000/2001*, p. 77, which presents unpublished tabulations of Peter Gottschalk based on the PSID.

Note: Unlike the table in **Exhibit 9a**, this table controls for age by comparing the income of the sample only to itself as it ages, not to the larger population.

It also tracks family income rather than individual income. This avoids the problem of attributing mobility to, for example, a teenager from a well off family who takes a minimum wage job during high school before joining the workforce in a more determined way after college.

Source: Di Tella, Rafael, "Inequality and the 'American Model'", Harvard Business School, March 2006.