The relation between lifespan and death rate

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Abstract

We prove that, in a steady state, the product of lifespan and death rate is one. This provides a benchmark for estimating the demographic trends in the future.

Initial draft. Comments welcome.

Over long term, few issues are as important as demographics of a society. To better study economics and social changes, it is essential to gain a deep understanding of demographic processes.

There are many parameters related to demographic issues, such as fertility, death rate and lifespan. These parameters are not independent to each other. But their precise relations are often not quantified. In this paper, we will study the relationship between death rate and lifespan. We will prove that under a steady state of population, the product of death rate and expected lifespan is one. This result provides a benchmark for the range of values of product of death rate and lifespan in real life. Then we will look into the values of the products of death rate and expected lifespan in different countries and the trend of their movements.

Death rate and lifespan in a steady state

From data, the relation between death rate and lifespan is varied. To simplify matter, we will assume a population in a steady state and study the relationship between death rate and lifespan in such situation.

We will first look at an extreme case. In one population, everyone lives to and die at eighty years old. Its lifespan is eighty years old. Every year, 1/80 of its population die when they reach the age of eighty. The product of death rate and lifespan is 1/80*80 = 1. Does this relation hold for general population in any steady state of population? This is what we are going to prove.

We assume the number of birth is n_0 every year. The numbers of people survive to age 1, 2, 3, … M, where M is the maximal lifespan, are n_1, n_2, n_3, … n_M respectively. Here $n_0 >= n_1 >= n_2 >= n_3 >= \ldots >= n_M$. Since the population is in a steady state, the total number of death each year is the same as the total number of birth each year, that is, n_0. Assume all the birth and death occur at the same time. The total population number is

$$S = \sum_{i=1}^{M} n_i$$

The death rate is n_0/S.

The expected lifespan is equal to

$$\frac{n_1}{n_0} + \frac{n_2}{n_0} + \cdots + \frac{n_M}{n_0} = \frac{n_1 + n_2 + \cdots + n_M}{n_0} = \frac{S}{n_0}$$

The product of death rate and expected lifespan is

$$\frac{n_0}{S} \times \frac{S}{n_0} = 1$$

This ends the proof. For a population in a steady state, the product of death rate and expected lifespan is 1.

The products of death rate and expected lifespan in different countries and their change over time

We will take a look at the products of death rate and expected lifespan in different countries and their change over time. World Health Organization has compiled health data of 183 countries. To gather a representative sample, we will use data from countries with life expectancy at birth ranked at number 1, 11, 21, … 181 in the year of 2019. There are nineteen countries altogether. These countries are

Rank by lifespan at 2019	Country
1	Japan
11	France
21	Austria
31	Chile
41	Ecuador
51	Algeria
61	United Arab Emirate
71	Libya
81	Belize
91	Venezuela
101	Ukraine
111	Honduras

121	Cambodia
131	Mauritania
141	Djibouti
151	Namibia
161	Angola
171	Botswana
181	Somalia

We download data from macrotrends.net. The data are from 1950 to 2100, in which 1950 to 2020 are statistical data, and 2021 to 2100 are projected numbers from UN.

In the following table, we list the product of death rate and lifespan of each country in every twenty years. Annual numbers are available for request from the author. Readers can also calculate the numbers easily from the data.

	1950	1970	1990	2010	2030	2050	2070	2090
Japan	0.60	0.48	0.52	0.76	1.11	1.30	1.47	1.29
France	0.88	0.78	0.73	0.70	0.82	1.00	1.02	1.01
Austria	0.80	0.90	0.83	0.75	0.86	1.07	1.11	1.09
Chile	0.74	0.56	0.41	0.44	0.60	0.85	1.06	1.20
Ecuador	0.97	0.70	0.43	0.38	0.43	0.57	0.75	0.94
Algeria	1.01	0.84	0.43	0.35	0.39	0.54	0.74	0.79
UAE	0.99	0.48	0.19	0.11	0.21	0.56	0.71	0.62
Libya	1.14	0.76	0.36	0.34	0.43	0.70	0.93	1.00
Belize	0.81	0.55	0.39	0.33	0.40	0.55	0.75	0.97
Venezuela	0.70	0.46	0.36	0.40	0.58	0.75	0.91	1.06
Ukraine	0.76	0.62	0.87	1.10	1.14	1.28	1.38	1.22
Honduras	1.07	0.79	0.46	0.34	0.36	0.50	0.73	0.99
Mauritania	0.91	0.79	0.59	0.52	0.44	0.46	0.52	0.59
Djibouti	0.88	0.75	0.61	0.55	0.50	0.67	0.87	0.97
Namibia	0.97	0.77	0.54	0.65	0.45	0.50	0.62	0.74
Angola	0.98	0.98	0.89	0.66	0.42	0.37	0.40	0.47
Botswana	0.94	0.74	0.53	0.57	0.44	0.56	0.74	0.87
Somalia	1.02	0.95	0.88	0.70	0.55	0.46	0.46	0.54

From the above table, we can make several observations. First, the product of death rate and lifespan is closer to one for most countries in 1950 than in 2010. The population structures in most countries were closer to a steady state in 1950 than in 2010. After the second world war, the popularization of petroleum-based technology, such as mass transportations, greatly improved the living conditions of people around the world. Death rates for the infants dropped sharply and population sizes increased rapidly. With increasing proportion of population concentrated at the young age, overall death rates become very low in many countries. This lowers the value of the product of death rate and lifespan.

Second, the expected product of death rate and lifespan will be closer to one for most countries in 2090 than in 2010. The population structures in most countries will be closer to a steady state in 2090 than in 2010. After the rapid population growth in the past century, the speed of growth has slowed down globally. In some countries, such as Japan, total population already has been in decline. There is a general trend toward a steady state. However, the paths of movements are quite diverse.

Third, the likely range of the value of the product of death rate and lifespan can be estimated from the minimal number and maximal number in the table. The minimal number is 0.11, obtained from UAE in 2010. The maximal number is 1.47, estimated for Japan in 2070. We can examine data of death rates and lifespans directly. But by looking at their products, we can gain a sense of where they are comparing with a steady state.

Concluding remarks

In this short paper, we prove that in a steady state of population, the product of death rate and lifespan is equal to one. This result suggests that for a population, the values of the product of death rate and lifespan may move around one. It may help us more precisely project the demographic dynamics into the future.

Take Japan's demographic dynamics as an example. In UN projection, the values of the product of death rate and lifespan will continue to rise around 2070 to the level of 1.47. Is it possible that the value may peak at the level lower than 1.47? This number looks pretty high. We will leave this and other demographic questions to future study.

The pull toward a steady state can be thought of a force in demographic dynamics. The magnitude of this force may be small compared to other forces. But this ever-present force will exert fundamental influence on demographic dynamics over long term.