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On the theoretical foundation of corporate finance ☆

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ABSTRACT

Modigliani and Miller theory forms the theoretical foundation of corporate finance. Yet Modigliani and Miller theory was derived from a very special case of cash flows. Weighted Average Cost of Capital (WACC), which is part of the Modigliani and Miller theory, plays a fundamental role in capital structure decision and asset valuation. Empirically, asset valuation calculated from cash flows discounted by WACC almost always differs from the sum of debt and equity values. We derive asset valuations for more general cashflows. Only when the debt equity ratio is constant over time, valuation by WACC is equal to the sum of debt and equity values.

1. Introduction

Corporate finance concerns about how financing decisions by corporations affect corporate values. The theoretical foundation of corporate finance largely consists of a single paper by Modigliani and Miller (1958) over sixty years ago. In that paper, Modigliani and Miller showed that in a perfect market, capital structure doesn't affect corporate value. There exists a cost of capital for an investment, independent of capital structure. They further derived a formula to calculate the cost of capital. It is the weighted average of costs of various financing methods. The calculated cost of capital is called Weighted Average Cost of Capital (WACC). With WACC, an investment project can be valued with free cash flows discounted by a single discount rate. The theoretical propositions of Modigliani and Miller (1958) are no longer controversial and have been accepted into standard economic theory (Miller, 1988; Myers, 2001). Today, discussion on capital structure and asset valuation focus on empirical issues with respect to various market imperfections (Graham and Leary, 2011).

A company or an investment project can be funded with equity and debt. The value of a company is the sum of its equity and debt. When the equity and the debt of a company are publicly traded, their values are directly observable by the public. After the development of Modigliani and Miller theory, the value of a company or an investment project can also be calculated by its cashflows discounted by WACC. If Modigliani and Miller theory is correct, the cashflow method should yield the same answer as the sum of values of the equity and the debt.

Empirically, different valuation methods often provide different valuations for the same company or investment project. One might think this could be due to market imperfection. Perfect capital markets are generally assumed to satisfy the following set of conditions.

1. Investors and firms can trade the same set of securities at competitive market prices equal to the present value of their future cash flows.
2. There are no taxes, transaction costs, or issuance costs associated with security trading.
3. A firm's financing decisions do not change the cash flows generated by its investments, nor do they reveal new information about them (Berk et al., 2019, p. 609).

Methods of discounting cashflows are not related to market perfectness. The following paragraph is a representative opinion about the application of Modigliani and Miller theory in investment valuation.

> In principle, the free cash flow approach is fully consistent with the dividend discount model and should provide the same estimate of intrinsic value ... This was demonstrated in two famous papers by Modigliani and Miller. However, in practice, you will find that values from these models may differ, sometimes substantially. This is due to the fact that in practice, analysts are

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always forced to make simplifying assumptions (Bodie et al., 2018, p. 598).

However, Modigliani and Miller only demonstrated the free cash flow approach is fully consistent with the dividend discount model when the cash flow is constant to perpetuity and capital structure is constant over time. What happens when the cash flows or capital structures of an investment are more general? In this paper, we will study a type of investment whose cashflows and capital structures is more general than the ones discussed in Modigliani and Miller (1958). In this type of investment, both expected coupon payments of the debt and expected dividend payments from equity will change at constant, but possibly different, rates over time. We derive the following result. Only when the rate of change of coupon payment of debt is equal to the rate of change of dividend payment of equity, valuation by WACC gives the correct answer of asset values. Equivalently, only when the debt equity ratio is constant over time, valuation by WACC gives the correct answer of asset values.

When the expected rate of change of dividend payout from an asset is higher (lower) than the rate of change of coupon payout, discounting by WACC will overvalue (undervalue) the asset. Sometimes the misvaluation can be substantial. Modigliani and Miller theory is not valid for general cash flows or general capital structures. The free cash flow approach is *not* consistent with the dividend discount model and will *not* provide the same estimate of intrinsic value, even *in principle*.

Modigliani and Miller theory forms the theoretical foundation of corporate finance. WACC has been used in most valuations in investment projects. Finance and economics are closely intertwined, in theory and in practice. Since Modigliani and Miller theory provides the wrong answers for asset valuations most of the time, all problems in research and practice in corporate finance and many problems in economic theories and policies need to be reconsidered. This vast task can only be tackled over time in the future. In the following, we will briefly touch on several theoretical and empirical implications.

First, the vast amount of empirical results in corporate finance can be reinterpreted, often with much simpler explanations. For example, it is well known that valuations from different models may differ, sometimes substantially. This is often attributed to the problem that in practice, analysts are always forced to make simplifying assumptions. (Bodie, Kane, Marcus, 2018) However, this is unlikely. The inputs, the expected cash flows, are high dimensional. The outputs, asset values, are low dimensional. Different models use the same inputs. If the models are theoretically equivalent, differences in valuations from different models can be adjusted to very small by calibrating inputs. Only when the models are mathematically non-equivalent, as we have shown, substantial differences in valuations can persist. Our calculations also show what causes the differences in valuations and the sizes of these differences with different valuation methods.

Second, many empirical patterns in investment theory can be understood with simpler explanations. For example, growth stocks are often overvalued and value stocks are often undervalued (Fama and French, 1993). Many explanations for this pattern are provided. For growth companies, which have high expected growth rates, discounting by WACC tends to over value their assets. For value companies, which have low expected growth rates, discounting by WACC tends to under value their assets. Misvaluation due to WACC may contribute to systematic misvaluation of growth and value companies.

The outputs of new investment projects generally are expected to grow for a period of time. This means many new investment projects or companies are overvalued with WACC. This overvaluation could be responsible in part for general level of stock market overvaluation (Fama and French, 2002). This also makes the stock markets prone to large scale corrections (Shiller, 2000).

Third, this work can offer further insight into how much misfit between empirical data and theory can be attributed to market imperfection. Modigliani and Miller theory contains at least to parts. The first part (Proposition I in MM) proves the irrelevance of capital structure to the firm value under the condition of market perfection. The second part (Proposition II in MM) provides the formula of WACC, which is used to calculate the value of a firm. Our work examines the validity of the second part (Proposition II of MM) of Modigliani and Miller theory for general cashflows and capital structures. Most works on the extension of Modigliani and Miller theory are about the first part of their theory on capital structure with imperfect competition and imperfect information.

The literature of imperfect competition and imperfect information has a long history. An Essay on Economic Theory, by Richard Cantillon, was first published in 1755. In it, Cantillon (1755) discussed the effect of imperfect competition and asymmetric information, which is sometimes called Cantillon effect. Before the development of Modigliani and Miller theory, a number of researchers have stated close equivalents of capital irrelevance theory "by appealing to intuition rather than by attempting a proof" (Modigliani and Miller, 1958). Among them, Williams (1938) made the following discussion about the relation between capital structure and firm value.

> 10. The law of the conservation of investment value.
>
> If the investment value of an enterprise as a whole is by definition the present worth of all its future distributions to security holders, whether on interest or dividend account, then this value in no wise depends on what the company's capitalization is. Clearly if a single individual or a single institutional investor owned all the bonds, stocks, and warrants issued by a corporation, it would not matter to this investor what the company's capitalization was. Any earnings collected as interest could not be collected as dividends. To such an individual it would be perfectly obvious that total interest- and dividend-paying power was in no wise dependent on the kind of securities issued to the company's owner. Furthermore, no change in the investment value of the enterprise as a whole would result from a change in its capitalization. Bonds could be retired with stock issues, or two classes of junior securities (i.e., common stock and warrants) could be combined into one, without changing the investment value of the company as a whole. Such constancy of investment value is analogous to the indestructibility of matter or energy; it leads us to speak of the Law of the Conservation of In- vestment Value, just as physicists speak of the Law of the Conservation of Matter, or the Law of the Conservation of Energy.
>
> Since market value does not usually conform exactly to investment value, no "conservation of market value" is to be found in general. Only to a rough extent do total market values remain the same regardless of capitalization. The exceptions in practice are important enough to afford many opportunities for profit by promoters and investment bankers. (Williams, 1938, p. 73)

Williams argued that the irrelevance of capital structure is a first approximation. He also recognized that good capital structure provides many profit opportunities in practice. Over time, more refined models of capital structure, such as pecking order theory, tradeoff theory and market timing theory, have been proposed and tested (Hovakimian et al., 2004). The literature of capital structure, mostly based on imperfect competition and imperfect information, has grown considerably (Kraus and Litzenberger, 1973; Myers and Majluf, 1984; Baker and Wurgler, 2002;

Titman, 2002; Faulkender and Petersen, 2006; Strebulaev, 2007; Frank and Goyal, 2009; Huang and Ritter, 2009; Lemmon and Zender, 2010; Knoll, 2018 and many others). Capital structure of firms is one of the most active research areas in finance. Many recent works have offered excellent literature review about the subject, which we will not repeat here.

How much of empirical patterns in capital structure can be attributed to various market imperfections? Some people, including Miller himself, question whether "imperfections", such as agency costs can account for the observed patterns of capital structure:

> The possibility that managers might let their own interest override that of the shareholders was something that Franco Modigliani and I were certainly aware of back in 1958 and through all our subsequent revisions and extensions; and we knew that anecdotal evidences of non-value maximizing behavior by under-diversified managers would always easy to come by. But we doubted that such nonoptimizing behavior would lead to *systematic* departures from the model. We believed that the stock holders would learn to solve, or at least greatly to mitigate any excessive risk aversion of their managers by appropriate compensation and incentive mechanisms. The stockholders, after all, could always persuade the managers to act more like stockholders by giving the managers stock or stock appreciation rights or stock options of any of the a number of kinds. Given the defenses available to stockholders to recapture value they believe belongs to them — including defenses such as large-shareholder influence and hostile takeover by outsiders — it is hard to believe that a sum as large as $150 billion a year would be left lying on the table. (Miller, 1998, p. 118).

Others often find the designs of many empirical tests are flawed (Molina, 2005). Our result offers a more accurate method of calculation to test the empirical validity of different hypotheses.

In general, if a so called imperfection of large scale persists for a long time, it often indicates a deep problem that is not well understood. The mathematical restriction in Modigliani Miller theory would be relatively easy to spot. However, over the years, many advanced methodologies have been developed to deal with many perceived market imperfections. Layers and layers of advanced methodologies built on top of the original Modigliani and Miller papers often obscure the true source of problems that may lie at lower and more fundamental levels. Our work studies the Modigliani and Miller theory at its most basic level. It shows that Modigliani and Miller theory systematically overvalue growth firms. The documented empirical pattern of systematic nonoptimizing behavior in the literature could be a result of imperfect theory instead of imperfect market.

Fourth, WACC should not be used in valuing expected future cash flows and making financing decisions. Currently, it is a standard practice to discount expected future cash flows with WACC. Financing decisions are often made according to the valuations based on WACC discounting. However, this practice often generates large amount errors in valuation. In particular, new investment projects often have high expected growth rates. This means new investment projects are systematically overvalued, sometimes substantially. This will lower efficiencies in resource distribution and have significant impacts in wealth distributions.

Fifth, structures and dynamics of systems are important for financial and economic research. Modigliani and Miller propositions are often considered as the conservation of risk. "The M&M propositions are the finance equivalents of conservation laws. What gets conserved in this case is the risk of the earning stream generated by the firm's operating assets." (Miller, 1990) Firms, like humans, don't have the same level of risk over time. When we buy life insurance at fifty years old, we pay more than at thirty years old. The discount rate used at any particular point of time represents the average risk of the entire life span. In the field of bond pricing, there is a terminology of convexity. This means that discount rate is not a linear factor. Weighted Average Cost of Capital is a linear combination of two (or more) discount rates. Intuitively, WACC will not provide accurate pricing for general cash flows or general capital structures. We provide a rigorous mathematical proof for this intuition. In the special case proved in Modigliani and Miller (1958) paper, in which risk is indeed constant over the perpetual life of the investment, WACC does yield correct answer. In general, an economic system will experience structural change over its life cycle. Its dynamics may not be able to be accurately described by linear models that dominate current economic research.

Sixth, this work offers a glimpse into the evolutionary dynamics of modern economic theories. WWII demonstrated the great power of mathematics in determining the outcomes of wars. The time period shortly after WWII was relatively conducive to new ideas and new theories. Most of the foundational works in modern economic and finance theories were developed in that period, such as Nash's work on game theory, Arrow Debreu model on general equilibrium, Markowitz portfolio theory and Sharpe Lintner CAPM model in investment theory, as well as Modigliani Miller theory on corporate finance. Over time, a large body of literature are built on these theoretical foundations. The academic system becomes increasingly senile. The academic environment becomes increasingly sterile. It has becoming increasingly difficult for groundbreaking ideas to germinate and grow in the academic fields.

Modigliani and Miller theory was a generalization from earlier financing model. They warned against drawing broad conclusions from special cases.

> This is merely one of a number of peculiarities of this special case on which, unfortunately, many writers have based their entire analysis. The reason for the preoccupation with this special case is far from clear to us. Certainly no one would suggest that it is the only empirically relevant case. Even if the case were in fact the most common, the theorist would be under an obligation to consider alternative assumptions. We suspect that in the last analysis, the popularity of the internal financing model will be found to reflect little more than its ease of manipulation combined with the failure to push the analysis far enough to disclose how special and how treacherous it really is. (Miller and Modigliani, 1961, P. 424).

Modigliani and Miller theory becomes a general financing model for investments. At the same time, it was based on very special cashflows. The investment assets are assumed to generate constant expected cashflows into perpetuity (Modigliani and Miller, 1958). Yet the theory is used to value general cashflows in practice. Our work is an extension of Modigliani and Miller theory into more general cash flows and capital structures. However, the academic environment today is very different from Modigliani and Miller's time.

Seventh, the new mathematical results will help stimulate open discussion on the theoretical foundation of corporate finance. Some researchers in corporate finance must be aware of the problems in asset valuation using WACC. In textbooks, numerical examples are often carefully constructed so debt equity ratios remain constant over time; WACC are sometimes given directly, instead of to be calculated from the formula, avoiding the inconsistencies between different valuation methods. At the same time, it is maintained that Modigliani and Miller theory is valid in all perfect markets. People are reluctant to discuss the issue openly. When we point out a fundamental problem in the theoretical foundation, without offering a replacement, we create a void beneath us. We avoid standing on void. We keep silent about the problem. I myself was aware of the restrictive nature of cash flows in Modigliani and

Miller theory long ago. However, I never discussed it publicly, or privately, until I constructed a mathematical proof of more general cash flows many years later.

The attempt to make Modigliani and Miller theory seems less restrictive is probably behind the research on optimal debt equity ratio. If there exists an optimal debt equity ratio for each company, a company will try to maintain such a constant debt-equity ratio. Then the emphasis on constant capital structure seems less restrictive. There are several problems for such an emphasis. First, a company at different stages of life cycle may prefer different capital structures (Damodaran, 2001). Second, companies may find themselves advantageous not to stick to their original target of debt equity ratio in certain market conditions. In a bull market when the equity becomes overvalued, a company may issue additional equity, instead of additional debt, as required by a constant ratio of debt to equity. Third, the designs of certain financial instruments indicate that maintaining a constant debt equity ratio is not important in corporate decisions. For example, when equity price rises, a company can force the holders of a convertible bond to convert into equity. This will further increase the weight of equity at a time of rising equity prices. Suppose many companies are serious about constant debt equity ratio. Some financial instruments that convert equity into debt in a rising equity market will be designed. The lack of such financial instruments, and the existence of convertible bond, shows that maintaining constant debt equity ratio is not of primary importance to most companies. Fourth, market prices change continuously but companies rebalance their capital structure only periodically. Researchers find most companies do not strictly enforce constant debt-equity ratio (Graham and Harvey, 2001).

There are other methods to deal with the differences of valuations related to WACC. One is to make WACC different for different years (Berk, DeMarzo and Stangeland, 2019; Fernandez, 2017). However, if WACC changes every year, some commonly used concepts in corporate finance, such as the cost of capital and the expected rate of return for projects, cease to apply. Furthermore, in practice we need to investigate if it is possible to determine the values of WACC of each year from market data.

This work is an update from an earlier paper (Chen, 2019). It is structured as follows. Section two provides a brief review of Modigliani and Miller propositions. Section three derives asset valuations by free cash flows discounted by WACC. With the exception of some special cases, discounting by WACC generally does not provide correct asset valuations. Section four concludes.

2. A brief review of Modigliani and Miller propositions

We will concern ourselves with Propositions I, II and III in Modigliani and Miller's 1958 paper. We will preserve Modigliani and Miller's original words and notations as much as possible in stating their propositions.

Proposition I: Let $\bar{X}$ stand for the expected return per year on the assets by the company. Denote by D the market value of the debts of the company; by S the market value of its common shares; by $V \equiv S + D$ the market value of all its securities or, as we shall say, the market value of the firm; and by ρ the expected rate of return appropriate to its risk. Then our Proposition I asserts that we must have in equilibrium:

$$V \equiv S + D = \frac{\bar{X}}{\rho} \tag{1}$$

The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ appropriate to its risk.

$\bar{X}$ is the expected cash flow of the firm that is available for distribution to shareholders and debtholders. With zero growth (which means no need to add new assets) and zero taxes, then $\bar{X}$ is sales minus costs minus depreciation. With no need to finance asset growth, $\bar{X}$ is also equal to coupon payments plus dividend payments.

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\bar{X}/V$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is

$$\frac{\bar{X}}{S+D} \equiv \frac{\bar{X}}{V} = \rho \tag{2}$$

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in a company whose capital structure includes some debt: the expected rate of return or yield, r_s, on the stock of the company is a linear function of leverage as follows:

$$r_s = \rho + (\rho - r_D)\frac{D}{S} \tag{3}$$

Where r_D is the yield of the debt of the company.

Proposition III. An investment project should be undertaken if and only if the expected rate of return of this project is as large as or larger than the cost of capital

The above are Propositions I, II and III in MM's paper. We can rearrange Eq. (3) to obtain

$$\rho = \frac{D}{D+S}r_D + \frac{S}{D+S}r_s = \frac{D}{V}r_D + \frac{S}{V}r_s \tag{4}$$

This means that the company's average cost of capital is the weighted average of the costs of its debt and its equity.

Propositions I and II were proved under the assumption that the expected return from the asset is constant over time and there are no taxes. However, formula (4) of WACC has since been used in literature and taught in textbooks as a general formula of cost of capital of firms. Some of the assumptions in the original MM 1958 paper had been relaxed (Stiglitz, 1969). Can formula (3) and (4) be extended to value assets with general cash flows? We will examine this issue in the next section.

3. Asset valuation by WACC with general cashflows and capital structures

The value of an asset is the sum of the values of its debt and equity. In corporate finance and investment literature, asset value is also defined as cash flows discounted by weighted average cost of capital (WACC) (Ross et al., 2013; Bodie, Kane, Marcus, 2018). Modigliani and Miller (1958) proved that when the expected return of an asset is constant over time, two definitions give the same result. We shall prove that in general, two definitions provide different valuations. The mathematical derivations are quite involved. We will make derivations in two cases. The second case is more general and more complex in derivation.

For the first case, suppose an asset is financed by a perpetual bond and an equity issue. The bond pays coupon amount c per unit time. The equity is expected to pay dividend amount d next time period. The amount of dividend is expected to change at a rate of g. The market value of the bond is D. Then the yield of the bond is

$$r_D = \frac{c}{D} \tag{5}$$

The market value of the equity is S. The discount rate on the dividends is r_s. Then

$$r_s = \frac{d}{S} + g \tag{6}$$

The asset value, V, is the sum of debt and equity. The value of an asset is also defined as total cash flows discounted by WACC.

Let V' represent the asset value calculated from this definition.

$$V' = \frac{c+d}{1+\rho} + \frac{c+d(1+g)}{(1+\rho)^2} + \frac{c+d(1+g)^2}{(1+\rho)^3} + \dots$$

$$= \frac{c}{1+\rho} + \frac{c}{(1+\rho)^2} + \frac{c}{(1+\rho)^3} + \dots + \frac{d}{1+\rho} + \frac{d(1+g)}{(1+\rho)^2} + \frac{d(1+g)^2}{(1+\rho)^3} + \dots$$

$$= \frac{c}{\rho} + \frac{d}{\rho-g}$$

$$= \frac{c}{\frac{D}{V}r_D + \frac{S}{V}r_s} + \frac{d}{\frac{D}{V}r_D + \frac{S}{V}r_s - g}$$

$$= V\left(\frac{c}{Dr_D + Sr_s} + \frac{d}{Dr_D + Sr_s - Vg}\right)$$

So

$$V' = V\left(\frac{c}{Dr_D + Sr_s} + \frac{d}{Dr_D + Sr_s - Vg}\right) \tag{7}$$

The difference between V' and V would be

$$V' - V = V\left(\frac{c}{Dr_D + Sr_s} + \frac{d}{Dr_D + Sr_s - Vg} - 1\right)$$

From (5) and (6), the above formula can be simplified into,

$$= V\left(\frac{c}{c+d+Sg} + \frac{d}{c+d+Sg-Vg} - 1\right)$$

$$= V\left(\frac{c}{c+d+Sg} + \frac{d}{c+d-Dg} - 1\right)$$

$$= V\frac{c(c+d-Dg) + d(c+d+Sg) - (c+d+Sg)(c+d-Dg)}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{-cSg + dDg + SDg^2}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{gSD\left(-\frac{c}{D} + \frac{d}{S} + g\right)}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{gSD(r_s - r_D)}{(c+d+Sg)(c+d-Dg)} \tag{8}$$

When g = 0, term (8) is equal to zero.

$V' = V$

In the special case when the expected growth rate of dividend payout is zero, WACC does provide correct valuation for asset cash flows, as proved by Modigliani and Miller (1958). From (8), when the growth rate is positive, $g > 0$,

$V' > V$

discounting by WACC will overvalue the asset. When the growth rate is negative, $g < 0$,

$V' < V$

discounting by WACC will undervalue the asset.

For the second case, an asset is also financed by a perpetual bond and an equity issue. The bond pays coupon amount c next time period. The amount of coupon will change at a rate of k. The equity is expected to pay dividend amount d next time period. The amount of dividend is expected to change at a rate of g. The market value of the bond is D. Then the yield of the bond is

$$r_D = \frac{c}{D} + k \tag{9}$$

From (9), the bond value D is

$$D = \frac{c}{r_D - k} \tag{10}$$

The market value of the equity is S. The discount rate on the dividends is r_s. Then

$$r_s = \frac{d}{S} + g \tag{11}$$

From (11), the equity value S is

$$S = \frac{d}{r_S - g} \tag{12}$$

The asset value, V, is the sum of debt and equity. The value of an asset is also defined as total cash flows discounted by WACC. Let V' represent the asset value calculated from this definition.

$$V' = \frac{c+d}{1+\rho} + \frac{c+d(1+g)}{(1+\rho)^2} + \frac{c+d(1+g)^2}{(1+\rho)^3} + \dots$$

$$= \frac{c}{1+\rho} + \frac{c(1+k)}{(1+\rho)^2} + \frac{c(1+k)^2}{(1+\rho)^3} + \dots + \frac{d}{1+\rho} + \frac{d(1+g)}{(1+\rho)^2} + \frac{d(1+g)^2}{(1+\rho)^3} + \dots$$

$$= \frac{c}{\rho-k} + \frac{d}{\rho-g} \tag{13}$$

From (4), (10), (12)

$$\rho = \frac{D}{D+S}r_D + \frac{S}{D+S}r_s$$

$$= \frac{\frac{c}{r_D-k}}{\frac{c}{r_D-k} + \frac{d}{r_S-g}}r_D + \frac{\frac{d}{r_S-g}}{\frac{c}{r_D-k} + \frac{d}{r_S-g}}r_s$$

$$= \frac{c(r_S-g)}{c(r_S-g) + d(r_D-k)}r_D + \frac{d(r_D-k)}{c(r_S-g) + d(r_D-k)}r_s$$

$$= \frac{c(r_S-g)r_D + d(r_D-k)r_s}{c(r_S-g) + d(r_D-k)} \tag{14}$$

From (13), (10), (12)

$$V' - V = V' - (D+S)$$

$$= \frac{c}{\rho-k} + \frac{d}{\rho-g} - \left(\frac{c}{r_D-k} + \frac{d}{r_S-g}\right)$$

$$= c\,\frac{r_D-\rho}{(\rho-k)(r_D-k)} + d\,\frac{r_S-\rho}{(\rho-g)(r_s-g)}$$

$$= \frac{c(r_D-\rho)(\rho-g)(r_s-g) + d(r_s-\rho)(\rho-k)(r_D-k)}{(\rho-k)(r_D-k)(\rho-g)(r_s-g)} \tag{15}$$

The denominator of (15) is larger than zero. We will only consider the numerator. It is

$$c(r_D-\rho)(\rho-g)(r_s-g) + d(r_s-\rho)(\rho-k)(r_D-k) \tag{16}$$

Plug (14) into (16), we have

$$c\left(r_D - \frac{c(r_S-g)r_D + d(r_D-k)r_s}{c(r_S-g) + d(r_D-k)}\right) \times \left(\frac{c(r_S-g)r_D + d(r_D-k)r_s}{c(r_S-g) + d(r_D-k)} - g\right)(r_s-g) +$$

$$d\left(r_S-\frac{c(r_S-g)r_D+d(r_D-k)r_s}{c(r_S-g)+d(r_D-k)}\right)\times\left(\frac{c(r_S-g)r_D+d(r_D-k)r_s}{c(r_S-g)+d(r_D-k)}-k\right)(r_D-k)$$

$$=c\frac{d(r_D-k)(r_D-r_s)}{c(r_S-g)+d(r_D-k)}\times\left(\frac{c(r_S-g)(r_D-g)+d(r_D-k)(r_s-g)}{c(r_S-g)+d(r_D-k)}\right)(r_s-g)+$$

$$d\left(\frac{c(r_S-g)(r_S-r_D)}{c(r_S-g)+d(r_D-k)}\right)\times\left(\frac{c(r_S-g)(r_D-k)+d(r_D-k)(r_s-k)}{c(r_S-g)+d(r_D-k)}\right)(r_D-k)$$

$$=cd(r_S-g)(r_D-k)(r_s-r_D)\left(\frac{1}{c(r_S-g)+d(r_D-k)}\right)^2$$

$$(-\ (\ c(r_D-g)+d(r_D-k))(r_s-g)+(\ c(r_S-g)+d(r_s-k))(r_D-k)) \tag{17}$$

The first row of (17) is positive. We will consider the second row. It is

$$-\ (\ c(r_D-g)+d(r_D-k))(r_s-g)+(\ c(r_S-g)+d(r_s-k))(r_D-k)$$

$$=c(r_S-g)(g-k)+d(g-k)(r_D-k)$$

$$=(c(r_S-g)+d(r_D-k))(g-k) \tag{18}$$

From (15) to (18),

$$V'-V=A(g-k) \tag{19}$$

Here A is a positive coefficient. More specifically,

$$V'-V=\frac{cd(r_S-g)(r_D-k)(r_s-r_D)(c(r_S-g)+d(r_D-k))}{(\rho-k)(r_D-k)(\rho-g)(r_s-g)(c(r_S-g)+d(r_D-k))^2}\ (g-k) \tag{20}$$

(19), as well as (20), is equal to zero only when g = k. In any other cases, (19), as well as (20), is not equal to zero. When g = k, debt equity ratio is constant over time. It is only when debt equity ratio is constant, asset value calculated as the sum of cashflows discounted by WACC is equal to the sum of debt and equity. In all other cases, asset value calculated as the sum of cashflows discounted by WACC is not equal to the sum of debt and equity.

From (19), when g > k,

$$V'>V$$

discounting by WACC will overvalue the asset. When g < k,

$$V'<V$$

discounting by WACC will undervalue the asset.

In general, asset values calculated from cash flows discounted by WACC are not equal to the sum of values of debt and equity. In practice, people sometimes choose a discount rate that equalizes two definitions of asset values and call it WACC (Brigham et al., 2017, P. 655). This is to solve for r in the equation

$$\sum_{i=1}^{\infty}\frac{Cashflows_i}{(1+r)^i}=D+S \tag{21}$$

and call r WACC. However, this r in general is not equal to the weighted average of costs of debt and equity,

$$\frac{D}{D+S}r_D+\frac{S}{D+S}r_s$$

It should not be called WACC.

4. Concluding remarks

Modigliani and Miller theory forms the foundation of corporate finance. The formula of Weighted Average Cost of Capital (WACC) was derived by Modigliani and Miller in their original 1958 paper. The formula has been used in all calculations of asset values that apply Modigliani and Miller theory. All results and conclusions in academic works and practical valuations are based on the assumption that WACC is theoretically sound. In this work, we point out that Modigliani and Miller derived the formula of WACC under the restrictive assumption of cashflows. They assumed the cash flows and capital structures are constant to perpetuity. Then we rigorously prove that under more general cashflow and capital structure conditions, discounting by WACC does not provide correct measure of asset value. Sometimes, the errors can be substantial. Because of this, fundamental issues in corporate finance and investment need to be reexamined.

There are many important problems in corporate finance: patterns of cash flows and funding preferences over different stages of life cycles, lifespans of investment and their relations to other factors, and many other problems (Chen, 2006, Treynor, 1996). However, in the current research environment, empirical results that contradict Modigliani and Miller theory always are attributed to market imperfection. Research on corporate finance won't flourish, until the research community acknowledge the obvious fact: Modigliani and Miller theory is not valid for general cash flows and general capital structures.

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