The Scope of Validity of Modigliani and Miller Propositions

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Abstract

Modigliani and Miller propositions form the theoretical foundation of corporate finance and asset valuation. They exert tremendous influence on financial research and financial practices. Yet Modigliani and Miller propositions were derived from very restrictive assumption of cash flows. We prove that their derivation is not valid for more general patterns of cash flows. Modigliani and Miller propositions, when applied to asset valuation, can generate systematic and sometimes substantial misevaluation. Specifically, using WACC to value growth firms will systematically overestimate the value of these firms.

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1. Introduction

Sixty years ago, Modigliani and Miller (1958) published some propositions on asset valuation and capital structures. These propositions have since become the theoretical foundation of corporate finance and investment valuation. Some important concepts, such as weighted average cost of capital (WACC), have sprouted from the Modigliani and Miller propositions. WACC has played a ubiquitous role in determining values and capital structures of companies and investment projects. The theoretical propositions of Modigliani and Miller (1958) are no longer controversial and have been accepted into standard economic theory (Miller, 1988; Myers, 2001). Today, discussion on capital structure and asset valuation focus on empirical issues (Graham and Leary, 2011).

Empirically, different valuation methods often provide different valuations for the same company or investment project. The following paragraph is a representative opinion about the application of Modigliani and Miller theory in investment valuation.

> In principle, the free cash flow approach is fully consistent with the dividend discount model and should provide the same estimate of intrinsic value ... This was demonstrated in two famous papers by Modigliani and Miller. However, in practice, you will find that values from these models may differ, sometimes substantially. This is due to the fact that in practice, analysts are always forced to make simplifying assumptions. (Bodie et al, 2015, p. 595)

In practice, valuations from different models may differ, sometimes substantially. But this is often attributed to practical issues rather than Modigliani and Miller theory. Analysts are always forced to make simplifying assumptions in practice. But the same assumptions are applied in different valuation models. It is not clear why valuations with the same assumptions could yield very different results, if different valuation models are theoretically equivalent. However, Modigliani and Miller propositions were proved under very restrictive simplifying assumptions (Modigliani and Miller, 1958). The investment assets are assumed to generate constant expected cash flows into perpetuity. Yet Modigliani and Miller theory is accepted to be valid for all types of investments in perfect market. Are Modigliani and Miller propositions really valid for more general cash flows?

To simplify discussion, we will study a type of investment whose cash flows is only slightly more general than the one discussed in Modigliani and Miller (1958). In this type of investment, debt payments are constant and expected dividend payments will change at a constant rate over time. We prove that when the expected growth rate of dividend payout from an asset is positive (negative), discounting by WACC will overvalue (undervalue) the asset. Sometimes the misevaluation can be substantial. Only in the very special case when the expected growth rate of dividend payout is zero, discounting by WACC does provide correct valuation for asset cash flows, as proved by Modigliani and Miller (1958). In general, the free cash flow approach is *not* consistent with the dividend discount model and will *not* provide the same estimate of intrinsic value, even *in principle*.

Researchers in corporate finance are aware of the difficulties in asset valuation using WACC. They adopt several methods to alleviate the problem. A common method is to assume companies maintain a constant debt-equity ratio (Berk, DeMarzo and Stangeland, 2015). In practice, a constant debt-equity ratio is difficult to maintain. First, market prices change continuously but companies rebalance their capital structure only periodically. Second, companies may find themselves advantageous not to stick to their original target of debt equity ratio in certain market conditions. In a bull market when the equity becomes overvalued, a company may issue additional equity, instead of additional debt, as required by a constant ratio of debt to equity. Third, a company at different stages of life cycle may prefer different capital structures (Damodaran, 2001). Due to these and other reasons, most companies do not strictly enforce constant debt-equity ratio (Graham and Harvey, 2001).

There are other methods to deal with the differences of valuations related to WACC. One is to make WACC different for different years (Berk, DeMarzo and Stangeland, 2015; Fernandez, 2017). However, if WACC changes every year, some commonly used concepts in corporate finance, such as the cost of capital and the expected rate of return for projects, cease to apply. Furthermore, in practice it will be impossible to determine the values of WACC of each year from market data.

This result has broad empirical and theoretical implications. Empirically, growth stocks are often overvalued and value stocks are often undervalued (Fama and French, 1993). Many explanations for this pattern are provided. For growth companies, which have high expected growth rates, discounting by WACC tends to over value their assets. For value companies, which have low expected growth rates, discounting by WACC tends to under value their assets. Misevaluation due to WACC may contribute to systematic misevaluation of growth and value companies.

The outputs of new investment projects generally are expected to grow for a period of time. This means many new investment projects or companies are overvalued with WACC. This overvaluation could be responsible in part for general level of stock market overvaluation (Fama and French, 2002). This also makes the stock markets prone to large scale corrections (Shiller, 2000).

Theoretically, Modigliani and Miller propositions were proved under very restrictive assumptions. Yet they are applied and taught in universities worldwide as truth in general conditions. Empirical results consistently contradict theoretical predictions. Always they are attributed to market imperfection. Thousands of papers are written on the empirical investigations of corporate finance and investment valuation related to Modigliani and Miller propositions. These empirical studies generally yield no systematic findings (Graham and Leary, 2011). Yet few investigate the theoretical foundation of Modigliani and Miller propositions. Our work shows that the current theoretical foundation yields systematic biases in investment valuation. Hope this will help stimulate further discussion on the theoretical foundations of finance and economics. After all, the standard finance and economic theories fit empirical data poorly (Fama and French, 2004; Galbraith, 2014).

2. A Brief Review of Modigliani and Miller Propositions

We will concern ourselves with Propositions I, II and III in Modigliani and Miller's 1958 paper. We will preserve Modigliani and Miller's original words and notations as much as possible in stating their propositions.

Proposition I: Let $\bar{X}$ stand for the expected return per year on the assets by the company. Denote by D the market value of the debts of the company; by S the market value of its common shares; by $V \equiv S + D$ the market value of all its securities or, as we shall say, the market value of the firm; and by ρ the expected rate of return appropriate to its risk. Then our Proposition I asserts that we must have in equilibrium:

$$V \equiv S + D = \frac{\bar{X}}{\rho} \qquad (1)$$

The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ appropriate to its risk.

$\bar{X}$ is the expected cash flow of the firm that is available for distribution to shareholders and debtholders. With zero growth (which means no need to add new assets) and zero taxes, then $\bar{X}$ is sales minus costs minus depreciation. With no need to finance asset growth, $\bar{X}$ is also equal to coupon payments plus dividend payments.

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\bar{X}/V$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is

$$\frac{\bar{X}}{S+D} \equiv \frac{\bar{X}}{V} = \rho \qquad (2)$$

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in a company whose capital structure includes some debt: the expected rate of return or yield, r_S, on the stock of the company is a linear function of leverage as follows:

$$r_S = \rho + (\rho - r_D)\frac{D}{S} \qquad (3)$$

Where r_D is the yield of the debt of the company.

Proposition III. An investment project should be undertaken if and only if the expected rate of return of this project is as large as or larger than the cost of capital

The above are Propositions I, II and III in MM's paper. We can rearrange equation (3) to obtain

$$\rho = \frac{D}{D+S}r_D + \frac{S}{D+S}r_S = \frac{D}{V}r_D + \frac{S}{V}r_S \qquad (4)$$

This means that the company's average cost of capital is the weighted average of the costs of its debt and its equity.

Propositions I and II were proved under the assumption that the expected return from the asset is constant over time and there are no taxes. However, formula (4) of WACC has since been used in literature and taught in textbooks as a general formula of cost of capital of firms. Some of the assumptions in the original MM 1958 paper had been relaxed (Stiglitz, 1969). Can formula (3) and (4) be extended to value assets with general cash flows? We will examine this issue in the next section.

3. Valuation of Asset Cash Flows with WACC

The value of an asset is the sum of the values of its debt and equity. In corporate finance and investment literature, asset value is also defined as cash flows discounted by weighted average cost of capital (WACC) (Ross, Westerfield, Jordan and Roberts, 2013; Bodie et al, 2015). Modigliani and Miller (1958) proved that when the expected return of an asset is constant over time, two definitions give the same result. We shall prove that in general, two definitions provide different valuations.

Suppose an asset is financed by a perpetual bond and an equity issue. The bond pays coupon amount c per unit time. The equity is expected to pay dividend amount d next time period. The amount of dividend is expected to grow at a rate of g. The market value of the bond is D. Then the yield of the bond is

$$r_D = \frac{c}{D} \tag{5}$$

The market value of the equity is S. The discount rate on the dividends is r_S. Then

$$r_s = \frac{d}{S} + g \tag{6}$$

The asset value, V, is the sum of debt and equity. The value of an asset is also defined as total cash flows discounted by WACC. Let V' represent the asset value calculated from this definition.

$$V' = \frac{c+d}{1+\rho} + \frac{c+d(1+g)}{(1+\rho)^2} + \frac{c+d(1+g)^2}{(1+\rho)^3} + \cdots$$

$$= \frac{c}{1+\rho} + \frac{c}{(1+\rho)^2} + \frac{c}{(1+\rho)^3} + \cdots + \frac{d}{1+\rho} + \frac{d(1+g)}{(1+\rho)^2} + \frac{d(1+g)^2}{(1+\rho)^3} + \cdots$$

$$= \frac{c}{\rho} + \frac{d}{\rho - g}$$

$$= \frac{c}{\frac{D}{V}r_D + \frac{S}{V}r_s} + \frac{d}{\frac{D}{V}r_D + \frac{S}{V}r_s - g}$$

$$= V\left(\frac{c}{Dr_D + Sr_s} + \frac{d}{Dr_D + Sr_s - Vg}\right)$$

So

$$V' = V\left(\frac{c}{Dr_D + Sr_s} + \frac{d}{Dr_D + Sr_s - Vg}\right) \tag{7}$$

The difference between $V' and\ V$ would be

$$V' - V = V\left(\frac{c}{Dr_D+Sr_S} + \frac{d}{Dr_D+Sr_S-Vg} - 1\right)$$

From (5) and (6), the above formula can be simplified into,

$$= V(\frac{c}{c+d+Sg} + \frac{d}{c+d+Sg-Vg} - 1)$$

$$= V\left(\frac{c}{c+d+Sg} + \frac{d}{c+d-Dg} - 1\right)$$

$$= V\frac{c(c+d-Dg)+d(c+d+Sg)-(c+d+Sg)(c+d-Dg)}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{-cSg+dDg+SDg^2}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{gSD\left(-\frac{c}{D}+\frac{d}{S}+g\right)}{(c+d+Sg)(c+d-Dg)}$$

$$= V\frac{gSD(r_S-r_D)}{(c+d+Sg)(c+d-Dg)} \qquad (8)$$

When g = 0, term (8) is equal to zero.

$$V' = V$$

In the special case when the expected growth rate of dividend payout is zero, WACC does provide correct valuation for asset cash flows, as proved by Modigliani and Miller (1958). From (8), when the growth rate is positive, g > 0,

$$V' > V$$

discounting by WACC will overvalue the asset. When the growth rate is negative, g < 0,

$$V' < V$$

discounting by WACC will undervalue the asset.

In general, asset values calculated from cash flows discounted by WACC are not equal to the sum of values of debt and equity. In practice, people sometimes choose a discount rate that equalizes two definitions of asset values and call it WACC (Brigham, Ehrhardt, Gessaroli and Nason, 2017, P. 655). This is to solve for *r* in the equation

$$\sum_{i=1}^{\infty} \frac{Cashflows_i}{(1+r)^i} = D + S \qquad (9)$$

and call *r* WACC. However, this *r* in general is not equal to the weighted average of costs of debt and equity,

$$\frac{D}{D+S} r_D + \frac{S}{D+S} r_S$$

It should not be called WACC.

4. Some numerical examples

We will apply the method from the previous section to some numerical examples.

Suppose a company is financed with a perpetual bond and common stock. The market value of bond is 100 million dollars and the market value of the equity is 100 million dollars as well. The asset value of the company, as the sum of debt and equity, is

100+100 = 200 million dollars.

The company is expected to distribute coupon amount to 3 million dollar and dividend amount to 3 million dollars next year. So the yield of the perpetual bond is 3% and the dividend ratio is 3%. Assume the growth rate of the dividend is 4% per year. The cost of equity is

Dividend yield + growth rate = 3%+4%=7%

and

$$WACC = \frac{D}{D+S} r_D + \frac{S}{D+S} r_s = \frac{100}{100+100} 3\% + \frac{100}{100+100} 7\% = 5\%$$

The sum of cash flows discounted by WACC is

$$\sum_{i=1}^{\infty} \frac{3+3(1+4\%)^{i-1}}{(1+5\%)^i} = \sum_{i=1}^{\infty} \frac{3}{(1+5\%)^i} + \sum_{i=1}^{\infty} \frac{3(1+4\%)^{i-1}}{(1+5\%)^i}$$

$$= \frac{3}{5\%} + \frac{3}{5\%-4\%} = 60 + 300 = 360\, million$$

This is the asset value calculated from total asset cashflows discounted by WACC. It is much higher than the asset value as the sum of debt and equity, which is 200 million.

In this example, the correct discount rate for total cash flows can be obtained by solving (9). As

$$\frac{3}{6\%} + \frac{3}{6\% - 4\%} = 50 + 150 = 200,$$

the discount rate is 6%. But this rate is not a weighted average of costs of debt and equity.

Next, we will study equity valuation of corporations. Equity values of corporations can be measured with different methods. It is understood, from Modigliani and Miller theory, that different methods should provide the same answer, at least in principle (Bodie et al, 2015). We will use a numerical example to calculate the value of a company from two different methods, dividend discount model and free cash flow from the assets.

Suppose the expected dividend of a company is 2 million next year. The dividend is expected to grow 15% per year for four years until year five. Then the expected annual dividend growth is 3% to perpetuity. The market value of the company's equity is 60 million. The company also issued a bond with principal of 30 million dollars. The coupon rate of the bond is 4% and the market value of the bond is 30 million dollars. The equity discount rate, r_s, implied from the market price of the equity, can be calculated from

$$\sum_{i=1}^{\infty} \frac{d_i}{(1+r_s)^i} = \sum_{i=1}^{5} \frac{d_i}{(1+r_s)^i} + \sum_{i=6}^{\infty} \frac{d_i}{(1+r_s)^i}$$

$$= \frac{d_1}{r_s - g_1}\left(1 - \frac{(1+g_1)^5}{(1+r_s)^5}\right) + \frac{d_6}{r_s - g_2}\frac{1}{(1+r_s)^5} = 60 \qquad (10)$$

Here $g_1 = 15\%, g_2 = 3\%, d_1 = 2, d_6 = 2 \times 1.15^4 \times 1.03 = 3.6.$ Substituting the above numbers into the (10), we obtain the calculated result of r_s to be 7.96%. With debt yield of 4%,

$$WACC = \frac{D}{D+S} r_D + \frac{S}{D+S} r_s = \frac{30}{30+60} 4\% + \frac{60}{30+60} 7.96\% = 6.64\%$$

The value of equity cashflows, when discounted by WACC, is

$$\sum_{i=1}^{\infty} \frac{d_i}{(1+WACC)^i} = \sum_{i=1}^{5} \frac{d_i}{(1+WACC)^i} + \sum_{i=6}^{\infty} \frac{d_i}{(1+WACC)^i}$$

$$= \frac{d_1}{WACC - g_1}\left(1 - \frac{(1+g_1)^5}{(1+WACC)^5}\right) + \frac{d_6}{WACC - g_2} \frac{1}{(1+WACC)^5} = 82.64$$

When we calculate value of debt cashflows, we will study two scenarios. First, we assume the company's debt will retire at year twenty. In this case, there are twenty total payments of interest and one payment of the principal when the debt matures. The total value of the debt cashflows discounted by WACC is 21.36 million dollars. Total value of asset cashflows is

$$82.64 + 21.36 = 104.00\ million\ dollars$$

Equity valuation based on this method is

$$104 - 30 = 74\ million\ dollars$$

Which is 14 million dollars higher than the value obtained from the dividend discount model.

Second, we will assume the company will roll over the debt at the same interest rate indefinitely. In this case, payments of interest extend to perpetuity. The total value of the debt cashflows discounted by WACC is 18.06 million dollars. Total value of asset cashflows is

$$82.64 + 18.06 = 100.7\ million\ dollars$$

Equity valuation based on this method is

$$100.7 - 30 = 70.7\ million\ dollars$$

Which is 10.7 million dollars higher than the value obtained from the dividend discount model. Whatever assumptions we make on debt payments in the future, the equity value is overestimated with WACC method.

Is it possible that we make unrealistic assumption about the values of equity discount rate? We solve for the case when DDM method and free cash flow method to assets yield the same answer. The only possibility is when equity discount rate is the same as the debt discount rate.

5. Concluding Remarks

The Modigliani and Miller propositions and WACC have been applied to many corporate finance and investment problems. In general, investment returns are discounted by the cost of capital, represented by WACC. The optimal capital structure, which is supposed to maximize firm value, is often defined as the debt equity ratio where WACC reaches its minimum. As the WACC discounting method does not provide correct measure of asset value, many fundamental issues in corporate finance and investment need to be reexamined.

The goal of financial management, as stated at the beginning of most textbooks, is to maximize shareholder value. After the introduction of the concept of WACC in the middle of the textbooks, the goal of financial management usually changes into the maximization of firm value, which is the sum of equity value and debt value. The goal of maximizing shareholder value and maximizing frim value are not always consistent with each other. Furthermore, calculating firm values with WACC will not provide correct valuation in general. In contrast, equity values and debt values usually have observable marker valuations. By calculating equity value and debt value separately, we can maintain a consistent approach of maximizing shareholder value throughout the whole teaching and research process. We can also determine the optimal capital structure from the process of maximizing shareholder value.

In this work, we only discuss one scenario where reality does not fit into assumptions of Modigliani and Miller propositions. There are many other scenarios. For example, the maturity dates of debts, which are fixed, are not the same as the maturity dates of equities, which are indefinite. There are many ways to improve the theoretical foundation of corporate finance and investment theory (Treynor, 1996; Chen, 2006)

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