A Scarcity Theory of Value
With Reflections on the Arrow Debreu Model

Jing Chen
School of Business
University of Northern British Columbia
Prince George, BC
Canada V2N 4Z9
Phone: 1-250-960-6480
Email: chenj@unbc.ca
Web: http://web.unbc.ca/~chenj/

James K. Galbraith
Lyndon B. Johnson School of Public Affairs
The University of Texas at Austin
Austin, TX 78713-8925, USA
Phone: 512-471-1244
Email: galbraith@mail.utexas.edu
Web: http://www.utexas.edu/lbj/faculty/james-galbraith/

Abstract

Value theory is the foundation of economic theory. Marginal utility theory value is the dominant value theory. Arrow Debreu model, which is built on the utility theory of value, forms the mathematical foundation of general equilibrium theory. But Arrow Debreu model doesn't describe economic reality well. We present a mathematical theory of the scarcity theory of value. It describes economic activities accurately. In particular, it explains the importance of monopoly in economic and social life.

1. Introduction

Value is the most fundamental concept in economics. Value theory forms the foundation of economic theory. In general, the foundation of a building, or a theory, is rarely touched most of the time. The same situation occurs to value theory in economics. Value theory occupies a peculiar position in the development of economic theory. Most of the time, it is of little concern to mainstream economists for it is generally thought to be completely resolved. But a major shift in economic thinking often begins with the emergence of new understanding about value. For example, Mill (1871) asserted that he had left nothing in the laws of value for any future economist to clear up, shortly before Jevons and others developed a marginal utility theory of value that became the core of neoclassical economics. Later Arrow and Debreu (1954) axiomatized value theory into a formal mathematical framework. After some further refinements, research in the area of value theory became essentially dormant again in the last several decades.

According to Walras (1873), the founder of general equilibrium theory, there are three main theories of value: utility theory, scarcity theory and labor theory. Walras argued that value is a function of scarcity. He said that it is too broad to define utility as value since many things with high utility, such as oxygen, are of no economic value. Likewise, it is too narrow to define labor as value, for many things that take little labor have high value. For example, although oil produced in Alberta takes much more labor than oil produced in Saudi Arabia, Alberta oil is not more expensive than Saudi oil. In conventional economic theories, additional concepts, such as rent, are invoked to explain this phenomenon. This makes the labor theory of value less general.

Economic theory is the study of scarce resources. Value theory is the foundation of economic theory. A scarcity theory of value is more aligned with the goal of economic theory than the marginal utility theory. Scarcity is also more fundamental than utility. When food is scarce, authority and mainstream media cannot lower its utility so people will not want food.

However, Walras' scarcity theory of value has not become the standard theory in economics. Instead, the marginal utility theory proposed by Jevons and others has since become the standard. Jevons (1871) presented a beautiful argument for the marginal utility theory in an economy of two persons. At equilibrium, marginal utilities are equalized. This equalized marginal utility is the exchange value between two goods. At the point where marginal utility is equalized, total utility reaches a maximum. This is a very appealing property. Further, Jevons was able to derive a beautiful mathematical theory for the marginal utility theory of value. He observed, "the Laws of Exchange are found to resemble the Laws of Equilibrium of a lever as determined by the principle of virtual velocities" (Jevons, 1871, p. vii). From this resemblance, he derived the mathematical formulation of marginal utility theory of value.

Jevons' derivation is very impressive. It impressed many later economists. Marginal utility theory of value has become the standard value theory over time. However, Jevons' derivation cannot be satisfactorily extended to an economy with many people. The intuition for this is quite clear. In a two person economy, prices can be established by bargain. But in some larger economies, such as in a supermarket, customers don't bargain individually. The prices are fixed by the stores. They are the same to every customer, independent of each customer's individual utility function.

Consequently, utility maximization, or optimality, in two person economy cannot be extended to many person economy. A new term, Pareto optimality, was created to replace optimality, the original concept.

Later, the marginal utility theory of value was formalized into a mathematical theory (Arrow-Debreu, 1954). Currently, it is the mainstream value theory. It is systematically represented by the classical book, *Theory of value; an axiomatic analysis of economic equilibrium* by Debreu (1959). As a mathematical theory, the Arrow-Debreu model is quite rigorous. As an economic theory, the Arrow-Debreu model has little relevance to reality (Shubik, 1961; Düppe, 2012). Arrow and Debreu were very aware of that. Many years after the Arrow-Debreu model became firmed established as the foundation of economic theory, Debreu (1991) offered a lengthy justification for the detachment of economic theory from economic reality. Arrow (1972) defended the model on the ground that there is no "genuine alternative model". However, Arrow himself had been actively looking for a "genuine alternative model" (Arrow, 1973).

In this paper, we will present a mathematical theory of a scarcity theory of value. It describes reality very accurately. The Arrow-Debreu theory is extremely challenging mathematically. The scarcity theory of value, on the other hand, is extremely simple mathematically. Value is represented by a logarithm function.

The rest of the paper is structured as follows. In Section 2, we discuss the Arrow-Debreu model. In Sections 3 and 4, we develop the mathematical theory of value-as-scarcity. The influence on value of factors such as scarcity, the number of producers and market size of a commodity can be understood naturally from the logarithm function. Since scarcity of resources, including human resources, is often regulated by institutional measures such as immigration laws and patent laws, the value of economic commodities is in great part a reflection of institutional structures. In next few sections, we apply this theory of value to understand regulation, competition, social structure and trade issues.

2. Some reflections on Arrow Debreu theory

The Arrow-Debreu model is a rigorous mathematical theory. It is an axiomatic theory. Debreu (1983) summarized the benefits of axiomatization.

> Axiomatization, by insisting on mathematical rigor, has repeatedly led economists to a deeper understanding of the problems they were studying, and to the use of mathematical techniques that fitted those problems better. It has established secure bases from which exploration could start in new directions. It has freed researchers from the necessity of questioning the work of their predecessors in every detail. Rigor undoubtedly fulfills an intellectual need of many contemporary economic theorists, who therefore seek it for its own sake, but it is also an attribute of a theory that is an effective thinking tool. Two other major attributes of an effective theory are simplicity and generality. Again, their aesthetic appeal suffices to make them desirable ends in themselves for the designer of a theory. But their value to the scientific community goes far beyond aesthetics. Simplicity makes a theory

> usable by a great number of research workers. Generality makes it applicable to a broad class of problems.
>
> In yet another manner, the axiomatization of economic theory has helped its practitioners by making available to them the superbly efficient language of mathematics. It has permitted them to communicate with each other, and to think, with a great economy of means. At the same time, the dialogue between economists and mathematicians has become more intense. (Debreu, 1983, P 99)

If a theory is a good description of reality, it will be beneficial that axiomatization "has freed researchers from the necessity of questioning the work of their predecessors in every detail." However, this is not the case in mainstream economics, as Debreu (1991) acknowledged several years later.

> Before the contemporary period of the past five decades, theoretical physics had been an inaccessible ideal toward which economic theory sometimes strove. During that period, this striving became a powerful stimulus in the mathematization of economic theory.
>
> The great theories of physics cover an immense range of phenomena with a supreme economy of expression. Of this, James Clerk Maxwell (1865) had given a notable example, as he described the electromagnetic field by means of eight equations at the time when mathematical economics was born and came of age in the middle of the 19th century …
>
> The benefits of that special relationship were large for both fields; but physics did not completely surrender to the embrace of mathematics and to its inherent compulsion toward logical rigor. The experimental results and the factual observations that are at the basis of physics, and which provide a constant check on its theoretical constructions, occasionally led its bold reasonings to violate knowingly the canons of mathematical deduction.
>
> In these directions, economic theory could not follow the role model offered by physical theory. Next to the most sumptuous scientific tool of physics, the Superconducting Super Collider whose construction cost is estimated to be on the order of $\$10^{10}$ (David P. Hamilton, 1990; see also *Science*, 5 October 1990), the experiments of economics look excessively frugal. Being denied a sufficiently secure experimental base, economic theory has to adhere to the rules of logical discourse and must renounce the facility of internal inconsistency. (Debreu, 1991)

Debreu pointed out that, "The great theories of physics cover an immense range of phenomena with a supreme economy of expression." He gave such an example. "Of this, James Clerk Maxwell (1865) had given a notable example, as he described the electromagnetic field by means of eight equations." Then he claimed, "In these directions, economic theory could not follow the role model offered by physical theory." The reason? "Next to the most sumptuous scientific tool of physics, the Superconducting Super Collider whose construction cost is estimated to be on the order of

10^{10} (David P. Hamilton, 1990; see also *Science*, 5 October 1990), the experiments of economics look excessively frugal."

Debreu seemed to forget the example of Maxwell he just mentioned. Maxwell's theory is based on the experiments of Faraday and others, which "look excessively frugal" from any standard. However, this frugality did not prevent Maxwell from developing "a Grand Unified Theory" (Debreu, 1991, p. 3) of a vast array of electromagnetic phenomena.

Debreu thought that economic theory is "Being denied a sufficiently secure experimental base". However, economic and financial activities are the most practiced and experimented activities in human societies. Economic and financial data are the most frequently recorded and scrutinized data.

Fischer Black, like Debreu, was trained as a mathematician. He had the opposite view about the amount of spending and the quality of research. Says Black:

> In my view, the basic problem with research in business (and economics) is not that it's too theoretical, or too mathematical, or too divorced from the real world (though all of these are indeed serious problems). The basic problem is that we have too much research, and the wrong kind of research, because governments, firms, foundations, and generous alumni support it.
>
> The way to create a more free marketplace of ideas was to stop subsidizing the production of new ideas. Professors should be paid for their teaching only, since the ones who are not interested in research will stop producing it, and the ones who are interested in research will do it anyway. The result will be a net gain for society. Fewer noise traders relative to information traders in the marketplace of ideas can be expected to increase efficiency in that market. (Mehrling, 2005, p. 301)

Far from being the stimulant of scientific breakthroughs, high spending is the best way to raise entry barrier, the best way to maintain the monopoly of the standard doctrine, the best way to stifle great ideas. Physics becomes sterile and could no more generate great ideas after its cost skyrocketed. Indeed, the Superconducting Super Collider mentioned by Debreu has since become the symbol of waste and extravagance of physics research. It was eventually scrapped by the US Congress. In physics, the age of Superconducting Super Collider has produced little, while the age of frugality one hundred years ago had generated many great scientific breakthroughs.

Similarly, economic theory becomes sterile because of its high costs. When there are so many grants to apply for, there is little incentive to develop great theories to "cover an immense range of phenomena with a supreme economy of expression". Indeed, there is every incentive to ignore and kill great theories to justify numerous expensive grants. Overall, when so much money is at stake, there is little incentive to seek truth.

In the Arrow-Debreu model, all possible production modes are available at the onset. It is a static model by design. A maximization process from producers and consumers leads to an equilibrium point. This gives a formal justification of the equilibrium theory. However, this is not how the real

world works. The Arrow-Debreu model has little relevance to reality. In a book review of Debreu's *Theory of Value*, Shubik wrote,

> it represents a tidying up of old work and problems which will not necessarily provide a stepping-stone for new work … I suggest that further development of economic theory will rest heavily upon the utilization and exploration of assumptions radically different from those employed in this work… Economics is not mathematics. Rigour is a necessary but not a sufficient condition for a valuable contribution to economic theory. (Shubik, 1961)

However, over the years, the Arrow-Debreu theory has gained widespread acceptance and has become the theoretical foundation of the standard economic theory. Why can this happen? There could be several reasons.

First, it is a mathematically rigorous theory. It gives prestige to the profession of economics. It also gives a perception that economic theory is established on the solid foundation. Second, the Arrow-Debreu theory is highly abstract. It is difficult to understand. Most economists simply have to believe it. This gives the Arrow-Debreu model, as well as the whole economic theory built on it, an aura of religion. People defend a religion more fiercely than science, especially a religion that gives us good jobs. The Arrow-Debreu theory also makes it extremely difficult for outsiders to challenge economic theory. Few would have the patience to take the tremendous effort to understand the model. Many years ago, J. K. Galbraith made a similar observation.

> And the very vigor of minor debate makes it possible to exclude as irrelevant, and without seeming to be unscientific or parochial, any challenge to the framework itself. Moreover, with time and aided by the debate, the accepted ideas become increasingly elaborate. They have a large literature, even a mystique. The defenders are able to say that the challengers of the conventional wisdom have not mastered their intricacies. Indeed these ideas can be appreciated only by a stable, orthodox, and patient man --- in brief, by someone who closely resembles the man of conventional wisdom. The conventional wisdom having been made more or less identical with sound scholarship, its position is virtually impregnable. The skeptic is disqualified by his very tendency to go brashly from the old to the new. (Galbraith, 1958, p 11)

3. The scarcity theory of value: A mathematical representation

In a scarcity theory of value, economic value is a function of scarcity. Scarcity can be defined as a probability measure P in a certain probability space. The value of any product shall satisfy the following properties:

(a) The value of two products should be higher than the value of each of them.

(b) If two products are independent, that is, if the two products are not substitutes or partial substitutes of each other, then the total value of the two products will be the sum of two products.

(c) The value of any product is non-negative.

The only mathematical functions that satisfy all of the above properties are of the form

$$V(P) = -\log_b P \qquad (1)$$

where b is a positive constant (Applebaum, 1996).

For those who are familiar with Shannon's theory of information, they will recognize that the function of value takes similar form as the value of information. Indeed this mathematical theory of scarcity was inspired by Shannon's information theory (Chen, 2005, 2015, 2018). In information theory, the base of the logarithm function is usually chosen to be two because there are two choices of code in information transmission, namely, 0 and 1 (Shannon, 1948). In economics, b, the base of the logarithm function represents the number of suppliers of a product or a service.

4. Main properties of the scarcity theory of value

In the following, we will discuss the properties of this simple analytical theory of value-as-scarcity.

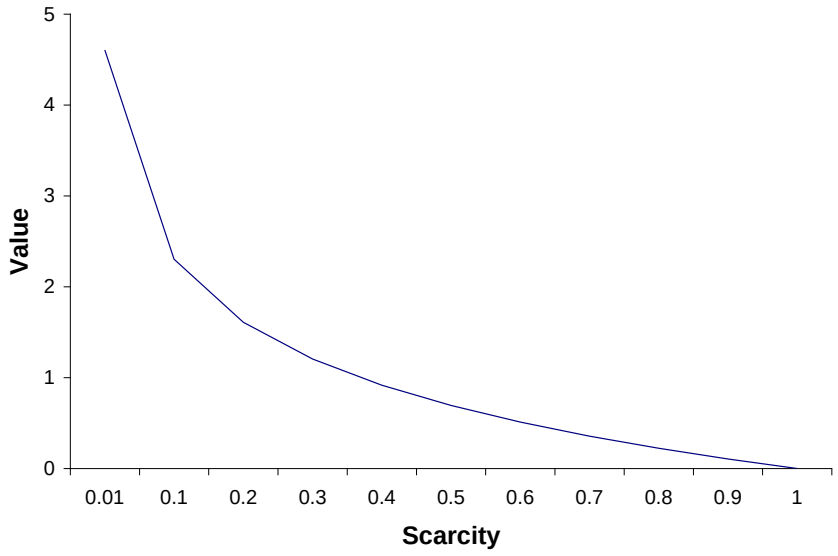

Figure 1. Value and scarcity

Scarcity and value

Figure 1 is a graph of Formula (1), which shows that value is an increasing function of scarcity. That is why diamonds are worth more than water in most circumstances. In extreme abundance, i.e., when P=1, -log P =0, the value of a given commodity is equal to zero, even if that commodity is very useful. For example, food is essential for survival. Most countries subsidize food production in various ways to guarantee the abundance of food. Cost of water usage in agriculture is much lower than industrial use. Land value on agriculture land is often not taxed. These subsidizing policies cause low economic value of agricultural products. This shows that economic value and social value can diverge.

Gold is mined on average at low concentrations and it takes much energy to grind up the rocks. Likewise silver, as compared to copper. In general, a scarce commodity takes more energy and labor to mine than an abundant commodity. The scarcity theory of value is highly consistent with the energy theory of value and the labor theory of value. An advantage of the scarcity theory of value is that it can be formulated as a mathematical theory easily.

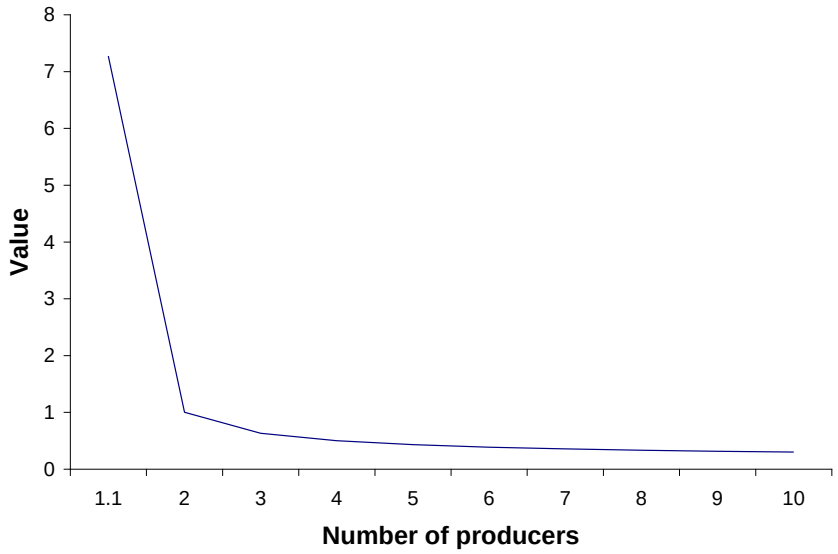

Figure 2. Value and the number of producers

Value and the number of producers or consumers

From Formula (1), value is inversely related to the number of producers of a given product. Figure 2 displays the relation between value and the number of producers. When the number of producers is small, the value of a product is high. That's why the products of monopolies and oligopolies are valued highly. If the base becomes one, i.e., absolute monopoly without substitution, value approaches infinity.

Governments have the monopoly on violence, legal decisions and taxation. Democratic societies maintain multiparty system and elections. This is to reduce the power, or the value, of governments. But monarchy or one party rule are more common in most of human history. Even in democratic societies, the power of governments, as monopolies, tend to grow over time. Today, governments' powers over citizens and businesses are much broader than (say) in the 19th century. Many important social activities, such as education and (in most countries) medicine, are under government control or influence.

In many countries, education is mostly funded by the government. Students can only go to one school for their elementary and secondary education. This lack of choice in education for students greatly increases the power and hence the value of educators.

The medical systems in some countries, such as Canada, don't allow patients to choose doctors and the types of treatments they can have. Instead, patients can only go to see one doctor, who decides what treatment a patient can get and who the patients can see. With this monopoly over patients, doctors gain extraordinary power and hence enjoy high incomes.

Successful religions, such as Judaism, Christianity and Islam, are monotheistic, while polytheistic religions, such as Buddhism, have difficulties to withstand the advance of monotheistic religions. Monotheism, with only one god, exerts much greater control over its believers than polytheism.

The number of providers of most economic goods depends on many factors. In the following, we give a brief discussion about the institutional structures that affect market entry and the number of suppliers for a given product.

Anti-trust regulations aim to prevent price fixing by existing providers of a service or product. They also intend to lower barriers to potential entry. By increasing the number of choices, both measures reduce the value of products, and hence the cost to consumers. For this reason, the value of a product will in general be lower in a more competitive market. Patent rights and commercial secrets legislation, on the other hand, grant monopoly power and discourage the diffusion of knowledge. Patent rights and monopoly power allow the holders to maintain high product prices. The IT industry has less strict patent protection than the biotech industry. As a result, IT develops much faster than biotech. In general, industries with more patent protection develop slower than industries with less patent protection. Technology often progresses very fast during war time, when patent laws are often ignored.

The quota system in trade policy forces the transfer of production technology from the dominant producer to other countries. Ultimately, the diffusion of technology and the increase of the number of producers will reduce the value of the imported goods. This will benefit the importing countries over the long term, instead of the loss suggested in standard literature.

The value of consumers is also negatively related to their numbers. When there is only one dominant customer, it can mostly dictate the terms of trade and hence would like to keep its monopoly power. Producers, on the other hand, would like to increase the number of their customers.

The relation between number of producers and value can help understand many commercial and social phenomena. Each printer manufacturer designs printers in a way that printer ink from other firms cannot operate well. Customers who buy printers from one company can use ink from only the same company. By restricting the choice from customers, producers can sell ink at higher price and obtain higher profits.

Unions form a monopoly of bargaining. With only a single unit of bargaining, a trade union is in a much stronger position than many individuals in bargaining with management. Unions are often formed in many stable professions, such as government employees and teachers. Professions such as physicians, often are certified by a single organization, which increases their monopoly value. Doctors' notes are famously illegible. When fewer people, especially patients, are informed, the value of the profession increases.

It is often difficult to determine the exact number of providers of a service empirically. Air travel in vast and thinly populated countries, such as Canada, where alternative modes of transportation are often very time consuming, provides a good testing ground. On March 10, 2005, Jetsgo, a Canadian airline, declared bankruptcy. There are three major operators in the air travel industry in Canada. They are Air Canada, WestJet and Jetsgo. There are regional carriers and international airlines competing for many routes. Most of the profits of airlines come from regional routes where competition is not intense. We can assume four providers for the air travel service for typical regional routes before Jetsgo declared bankruptcy. From (1), the value of each airline can be represented as

$$-\log_4 P \qquad \text{and} \qquad -\log_3 P$$

before and after Jetsgo declared bankruptcy. The change of value is therefore

$$(-\log_3 P)/(-\log_4 P) - 1 = \log_3 4 - 1 = 0.262$$

Jetsgo declared bankruptcy at the evening of March 10, 2005, after the market closed. The closing prices of stocks of WestJet and Air Canada at March 10 and 11 are 11.17, 15.6 and 32.19, 37 respectively. The price changes are

$$15.6/11.17 - 1 = 0.397 \qquad \text{for WestJet}$$

and

$$37/32.19 - 1 = 0.149 \qquad \text{for Air Canada}$$

respectively. The average change of price is

$$(0.397 + 0.149)/2 = 0.273$$

which is very close to the theoretical prediction of 0.262.

Some theoretical and empirical results can be further refined. For example, this theory does not distinguish the sizes of different providers of a service. The refinement of the theory is left to the future research.

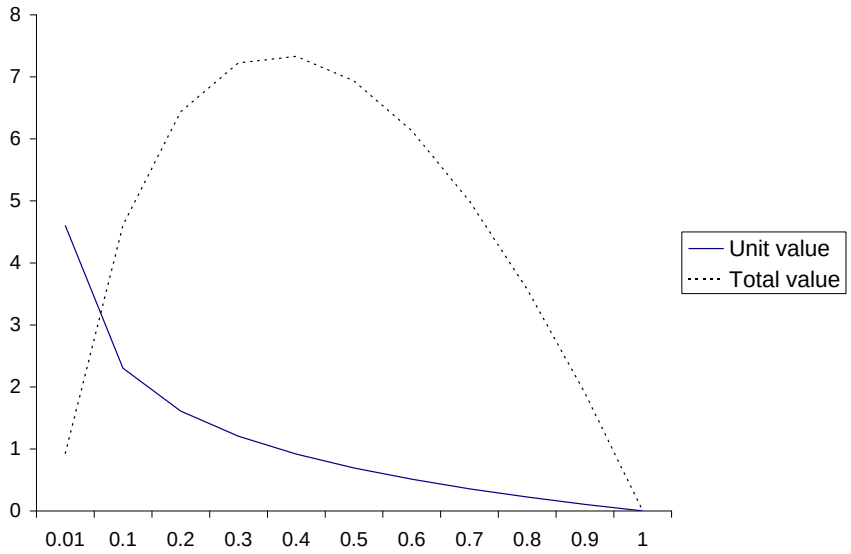

Figure 3. The unit value and total value of a product with respect to scarcity

Market size, product life cycle and product value

Suppose the potential market size of a product is M. The percentage of people who already have the product is P. Then the unit value of the product is

$$-\log P \qquad (2)$$

Since the number of people who have bought the product is MP, The total value of the product is

$$MP(-\log P) \qquad (3)$$

From (3), the value of a product is higher with a larger market size. Figure 3 is the graph of unit value and total value of a product with respect to its abundance. From Figure 3, we can explore the relation between the value of a product and the product life cycle. When a product is new and scarce, the unit value is high. Its total value is low. As the production increases, the total value will increase as the unit value decreases. When the production quantity is over a certain level, however, the total value of a product will start to decrease as well. Intuitively, this is easy to understand. The market values of manufacturers of mature products are generally low, although the production processes are very efficient. This observation shows that efficiency is not equivalent to value.

The above discussion shows that the implications of identifying value with the scarcity are highly consistent with our intuitive understanding of economic value. It should be noted that in economic processes, a final product embodies many different kind of scarcities: labor, raw materials and equipment. A detailed analysis of the value of a particular product will be much more involved.

While economic values of commodities are highly correlated with the level of physical scarcity, they are not identical for many reasons. One reason is that scarcity of a commodity is regulated by institutional structures. For example, the value of an invention is influenced by how long and how broad patent protection is granted. The value of a patent is higher in a system where patents are valid for twenty years than one where patents are valid for ten years. If patent protection is defined more broadly, the market is larger and the value of an invention is higher. Economic value, as a function of scarcity, is to a great extent regulated by institutional structures. Among all the institutional measures that regulate scarcity, the most important regulation is the immigration laws that regulate the scarcity of the labor force, which makes persistent large wage differentials across regions possible. Wage differentials can persist for other reasons such as relocation costs, or differences in cost-of-living. Wages in the cities are higher than in the countryside in the same country, with no legal barriers to migration. But these wage differentials are relative minor compared with wage differentials where legal barrier to migration is high.

This value theory can be applied to understand many economic activities. In next several sections, we will present some of the applications.

5. Regulations and other entry barriers in market competition

Regulations and other entry barriers increase fixed costs. Higher fixed cost reduces the number of businesses in an industry, which increases the value of their products or services. We will analyze how the increase of fixed cost affects the return of an industry. Suppose the market size of an industry is M, scarcity is p, the number of businesses in the industry is b. Then the unit value for the product is

$$-\log_b p$$

The fixed cost for each business is K, variable cost is C. Assume each business gets the same amount of revenue. The revenue and total cost for each business are

$$\frac{Mp}{b}(-\log_b p) \ \ and \ \ K + C\frac{Mp}{b}(-\log_b p)$$

respectively. The return for each business is

$$\frac{\frac{Mp}{b}(-\log_b p)}{K+C\frac{Mp}{b}(-\log_b p)} - 1 \qquad (4)$$

Suppose M = 1000, p = 0.4, b = 3. The fixed cost of each business is 35 and the variable cost of each business is 60% of the revenue. The rate of return for each business is

$$\frac{\frac{1000 \times 0.4}{3}(-\log_3 0.4)}{35 + 0.6\frac{1000 \times 0.4}{3}(-\log_3 0.4)} - 1 = 0.09$$

This rate of return is not very high. Now a business persuades the government to increase regulatory measure on this industry. As a result, the fixed cost is increased to 50. Assume other parameters remain the same. The new rate of return for each business, calculated from (4), becomes negative. If the rate of return becomes negative, one business, usually the financially weak one, will drop off the market. Suppose now there are only two businesses in the industry. Assume other parameters remain the same. The new rate of return for each remaining business is

$$\frac{\frac{1000\times0.4}{2}(-\log_2 0.4)}{50+0.6\frac{1000\times0.4}{2}(-\log_2 0.4)} - 1 = 0.27$$

This is much higher than the previous rate of return. Financially strong companies can use regulatory tools to increase fixed cost. It can reduce the number of competitors and help remaining players achieve high rate of return.

In neoclassical economics, regulation is justified when there is a "market failure". From the above analysis, regulations are largely driven by industries themselves to keep a high rate of return. Regulations are mostly initiated by stronger players in an industry for they are in better positions to spend the extra fixed costs. The theory also explains why biological and chemical weapons are banned by international treaties while nuclear weapons, which can cause much more destruction than chemical weapons, are not. Biological and chemical weapons, which are sometimes called poor men's nuclear weapons, are cheap to make. If these weapons are not banned, many people can make them, which will reduce the value of weapons of mass destruction. To maintain the high value of such weapons, international treaties, which are generally initiated by leading political powers, banned those weapons of mass destruction that are cheap to make.

6. **The compartmentation of the society**

Our society has become increasingly compartmented. Academic research is divided into increasingly narrower disciplines. Inside each discipline, researchers can ignore the knowledge from other disciplines. Even when certain knowledge is well known in other disciplines, researchers can discover the same "new" ideas again and again, as long as they dutifully cite the essential references in their own disciplines. When theories in different disciplines contradict each other, few would bother to find out where the problems are. When outsiders point out mistakes made by the emperors in a field, they are resolutely ignored. Why our societies are so compartmentalized? There are many official explanations. Let's look at it from the value perspective.

Suppose there are four service providers in one type of business. Originally, anyone can seek service from one of the four service providers. Later, the market is subdivided into two sections. Each section has half the population. Each section has two service providers. Residents in each section can only seek service from providers in their own section. What are the values of each service provider before and after the compartmentation of the market?

Let the scarcity of the service to be P. The compartmentation doesn't change the total population and the total number of the service providers. It doesn't change the scarcity of the service. Before compartmentation, there are four service providers. The value of the service is

$$-log_4 P$$

After compartmentation, there are two service providers in each section. The value of the service is

$$-log_2 P \ = 2(-log_4 P)$$

The value of the service doubled after market compartmentation. This is due to the reduction of choice for each customer. Since market compartmentation greatly increases the value of service providers, there is a strong incentive for a market to become increasingly segmented. We will use an example in economic research to illustrate the consequence of compartmentation in academic research.

The success of Shannon's information theory has stimulated the interest of many people. Arrow attempted to develop an information theory of value. After examining Shannon's idea, Arrow concluded,

"the well-known Shannon measure which has been so useful in communications engineering is not in general appropriate for economic analysis because it gives no weight to the value of the information. If beforehand a large manufacturer regards it as equally likely whether the price of his product will go up or down, then learning which is true conveys no more information, in the Shannon sense, than observing the toss of a fair coin" (Arrow, 1973, p. 138).

The Shannon measure actually measures the weight of information. For example, *N* symbols with identical Shannon measure carry *N* times more information than a single symbol (Shannon, 1948). That is why a long message carries more information than a single word. Similarly, the value of the information about the future price is higher to a large manufacturer than to a small manufacturer, other things being equal.

We can also understand the issue of weight from the perspective of physics. In physics, there are two types of quantity. The first type is called intensive quantity, such as temperature and pressure. The second type is called extensive quantity, such as volume and mass. You can double the mass of bricks if you put two bricks together. But you can't double the temperature of bricks if you put two bricks together, for temperature is an intensive, not an extensive quantity. Entropy is an extensive quantity. It naturally carries weight.

If Arrow didn't make such a simple mistake, he would have found a "genuine alternative model" of value theory long ago. Since entropy is the measure of scarcity in physics, an entropy theory of value is naturally a scarcity theory of value. If the academic world were less compartmentalized, someone from information theory, from physics, or anyone who has read Shannon's paper, could have corrected the mistake. The mistake had been pointed out repeatedly (Chen, 2005, 2015, 2018). However, the establishment resolutely ignore the problem. In a highly compartmentalized academia, the silence of authority is silencing any fundamental new ideas from outsiders.

7. Free trade: Winners and losers

Trade policies can be open or restricted. Access to the market can be easy or difficult. What are their effects? Does free trade increase the total wealth? Who are the winners and losers in trade? In general, trade occurs between regions with differential abundance of a commodity, which could be due to differential concentration of natural resources or the capacity of some manufacturing technology. Most oil exports occur in several countries. High tech industries are highly concentrated in Silicon Valley. To examine the quantitative impacts of a trade policy, we will look at a two region case and calculate a numerical example. (The proof of the general case is available from the author.) Let the market sizes of two regions be 100 and 1000 respectively, with resource concentration of 0.9 and 0.2. This indicates that the smaller region is abundant in a particular commodity. Suppose two regions are segregated. Then the commodity prices at two regions are

$$-\ln(0.9) = 0.11 \quad \text{and} \quad -\ln(0.2) = 1.61 \tag{5}$$

The commodity price in the abundant area is much cheaper. The total values of the commodity in two regions are

$$100*0.9*(-\ln(0.9)) = 9.48 \quad \text{and} \quad 1000*0.2*(-\ln(0.2)) = 321.89 \tag{6}$$

The global total value of the commodity is

$$9.48+321.49 = 331.37 \tag{7}$$

When two regions are integrated into a free trade zone, the global scarcity of the commodity is

$$(100*0.9+1000*0.2)/(100+1000) = 0.26 \quad (8)$$

The new price of the commodity is

$$-\ln(0.26) = 1.33 \quad (9)$$

The global value of the commodity is

$$1100*0.26*(-\ln(0.26)) = 386.62 \quad (10)$$

The total value of the commodity in the resource rich region is

$$100*0.9*(-\ln(0.26) = 119.99 \quad (11)$$

The total value of the commodity in the resource poor region is

$$1000*0.2*(-\ln(0.26) = 266.64 \quad (12)$$

In the following, we analyze the winners and the losers in a free trade economy. First, free trade increases the total value of a product. From (10), the global value of the commodity in a free trade environment is 386.62. From (7), the global value of the commodity in a segregated economy is 331.37, which is lower than the global value in a free trade environment.

Second, with free trade, producers in resource rich countries will increase their wealth while producers in resource poor countries will reduce their wealth. from (11) and (6), total value from the resource rich region in the free trade environment is 119.99, which is higher than 9.48, the total value in a segregated economy. That is why producers from the resource rich region will promote free trade. From (12) and (6), total value from the resource poor region in the free trade environment is 266.64, which is lower than 321.89, the total value in a segregated economy. That is why producers from the resource poor region will resist free trade.

Third, with free trade, consumers in resource rich countries will pay higher prices while consumers in resource poor countries will pay lower prices. From (5) and (9), the unit value of the commodity in a free trade environment is 1.33, which is higher than 0.11, the unit value of the commodity in the resource rich region and lower than 1.61, the unit value of the commodity in the resource poor region in a segregated economy. Ordinary consumers in a resource rich country who do not receive income from the resource industry will resist free trade. Ordinary consumers in a resource poor country who do not receive income from the resource industry will welcome free trade.

Fourth, the magnitude of impact to small and large regions are different. From (11) and (6), for the small region, the change of commodity value is from 9.48 to 119.99, which is very high. From (12) and (6), for the large region, the change of commodity value is from 321.89 to 266.64, which is moderate. As a result, small regions have stronger incentive to influence trade policies, although

large regions are often more powerful. For example, Canada charges a 270% tariff on import dairy products to deter US dairy imports. By comparison, the US charges a 27% tariff on Canadian lumber imports. In general, small social groups often have stronger internal cohesion than large social groups. Empires are often ruled by minority groups. The most powerful people and most wealthy people in a country are often from social minorities.

The main calculation results are summarized in Table 1.

	Segregated market		Integrated market
	Resource poor region	Resource rich region	
Market size	1000	100	1100
Scarcity	0.2	0.9	0.26
Unit price	1.61	0.11	1.33
Value in segregation	321.89	9.48	331.37 (sum in segregation)
Value in integration	266.64	120	386.62
Difference in value	-55.25	110.51	55.25

Table 1: Summary of value changes with market integration

There are two major price indices in the crude oil market: WTI and Brent. Historically, WTI and Brent crude oil prices were very close. However, WTI traded at a deep discount to Brent in recent years as Alberta increased its oil output, most of which was sold in the US. In an attempt to sell more oil at the international price, proposals were made to build or expand several oil pipelines to the coastal area so Alberta oil can be supplied to the international market. This would increase the value of Alberta oil products. Canada produces about three million barrels of crude oil per day. Canadian oil is often sold several dollars per barrel below the international price. Every year the Canadian oil industry loses several billion dollars from this price differential; equivalently, customers of the Canadian oil gain several billion dollars per year from the current situation. From the above analysis, it is easy to understand why there is so much negative publicity and disruption around the pipeline projects.

8. The detailed effects of trade tariffs

Next, we will analyze how trade tariff affects import and export countries. From our value theory, product value is a function of scarcity. Tariff policy can often significantly influence output quantity and hence product value, especially when a certain commodity has one big producer and one big consumer. For example, Canada is a big producer of softwood lumber while USA is a big consumer. From value theory, the value of lumber market is represented by VP(-*ln*P), where *P* is

the proportion of lumber that is on the market. Assume V, the total volume of the forest, is 10000. A consumer country will benefit from a trade policy that increases the production of lumber since it will reduce the value of imported lumber.

Suppose the cost structure of the lumber industry is the following. The total fixed cost in lumber production in country C is 100. The variable cost is 55% of product value. The total value of the lumber products is VP(-*ln*P) and the total cost of production is 100+0.55*V*P*(-lnP). Suppose every year, 1% of the all lumber is harvested. The profit on lumber production is equal to revenue minus total cost

$$\begin{aligned} & -VP\ln P-(100+0.55*(-VP\ln P)) \\ & =-10000*0.01*\ln(0.01) \\ & \quad -(100+0.55*(-10000*0.01*\ln(0.01))) \\ & =107 \end{aligned}$$

In 2001, the USA imposed a 27% import duty on lumber from Canada. If the volume of production remained at the same level, the profit for lumber production would be

$$\begin{aligned} & -VP\ln P*(1-0.27)-(100+0.55*(-VP\ln P)) \\ & =-10000*0.01*\ln(0.01)*(1-0.27) \\ & \quad -(100+0.55*(-10000*0.01*\ln(0.01))) \\ & =-17 \end{aligned}$$

which means that the lumber industry would lose money. Production of lumber had to be increased to avoid loss. If the production level is increased to P = 1.5%, the profit for the lumber industry will become

$$\begin{aligned} & -VP\ln P*(1-0.27)-(100+0.55*(-VP\ln P)) \\ & =-10000*0.015*\ln(0.015)*(1-0.27) \\ & \quad -(100+0.55*(-10000*0.015*\ln(0.015))) \\ & =13 \end{aligned}$$

As the production is increased from 1.0% of the total reserve to 1.5%, the unit value of lumber is decreased from –ln(0.01)= 4.6 to –ln(0.015) = 4.2. USA collected a 27% tariff on lumber import and enjoyed lower price on lumber. Table 1 gives a summary of softwood lumber futures prices, annual production from Canada, revenues and profits from Canfor, Canada's largest softwood producer, in 2000 and 2002, one year before and after USA imposed the 27% tariff on softwood lumber import from Canada.

The data confirm the theoretical prediction that after the tariff, production increased, prices dropped, and corporate profits from lumber producers tumbled. This shows that tariffs are an effective way to shift wealth from producing countries to consuming countries, and contradicts the standard theory that tariffs hurt importing countries by imposing higher prices for consumers.

	2000	2002
Softwood lumber futures price (January closing)	346.6	268.7
Production (thousands of cubic meters)	68557	71989
Canfor revenue (millions of dollars)	2265.9	2112.3
Canfor profit (millions of dollars)	125.6	11.5

Table 2. Summary statistics of softwood lumber futures price, annual production from Canada, revenues and profits from Canfor. Sources of data: CME, indexmundi, Canfor annual reports

From the theoretical analysis, as well the data in Table 2, trade policies have huge effects on the distribution of wealth across borders, and this also greatly influences the distribution of jobs across borders. This is why trade policies are such an emotional issue over history.

The scarcity of a commodity is influenced by the market size. For Canadian lumber, the market size is very much determined by the US housing market, which is much larger than the Canadian market. The market size is also greatly affected by transportation costs. For example, petroleum is relatively light compared with coal for the same amount of energy. Therefore, petroleum is a global commodity while coal is much less so. Lumber is six times heavier than coal as a fuel. Hence the market for wood as a fuel is highly localized. But the market for wood as lumber, which is higher priced than fuel, is much larger. Still, the increasing cost of oil decreases the size of the lumber market. Not only do transportation costs increase, but also higher energy prices can make constructing a home more expensive.

9. Concluding remarks

Jevons developed a simple and elegant mathematical model for the marginal utility theory of value in a two person economy. That was a great appeal of this theory. But when the theory was generalized into multi person economy, it encountered many difficulties. These difficulties show that the marginal utility theory of value could not describe reality accurately. The utility theory of value eventually morphed into the Arrow-Debreu model. It is rigorous mathematically. Yet the Arrow-Debreu model has little relevance to economic reality.

The mathematical model developed from scarcity theory of value describes reality accurately. It is also very simple. It greatly clarifies our understanding of a broad range of social and economic phenomena.

Some might argue that the scarcity theory of value offers less answers than the utility theory of value. It doesn't prove the existence of the equilibrium state while the utility theory does. It doesn't prove the existence of the equilibrium state because equilibrium state doesn't exist in real life. We are not at the "end of history". Western countries are not "developed" countries. Most western

countries have far below replacement fertility rate. Most western countries have aging populations. Most western countries, with aging populations, are very vulnerable to pandemics and other disturbances. Equilibrium theory gives us a false sense of security.

A scarcity theory of value doesn't assert the Pareto optimality of any system, while the utility theory does. For example, in North Korea, the Kim family enjoys great power and privilege. Any change of the social system, no matter how beneficial to the majority of the people, will harm the Kim family. Thus, the North Korean system is Pareto optimal! In a world with more than seven billion people, a social system that benefits a single person at the cost of more than seven billion people is Pareto optimal. Indeed, any existing or once-existed social system benefits the ruling elite and is actively defended by the ruling elite. They are all Pareto optimal. Whatever the original intention for the early developers of the concept of Pareto optimality, it has become the justifier of the status quo. It has become the defender of the most oppressive social systems in the world.

A scarcity theory lacks the innate appeal of Pareto optimality and general equilibrium. Nothing about it suggests that the beautiful system it describes will last forever. But... such a theory has an offsetting advantage. It describes the world as it is, and not as the ruling elites would like to imagine.

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