Remarks on the Chen–Galbraith paper

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In this very brief note, I would like to make a few remarks on the working paper by Chen and Galbraith [1]. The purpose of the paper is to formulate a theory of economic production by taking the higher vantage point of formulating principles that are applicable not only to economics but other areas as well such as biology and sociology. As such, what the paper proposes are principles that are intended to be universally applicable to a broad spectrum of complex systems. I will confine my remarks here to two issues.

I. COMPLEXITY AND FIXED COST

Chen and Galbraith [1] make the following interesting observation in the subsection of their paper titled "fixed cost and uncertainty":

> "By calculating variable costs from (5), we find that, as fixed costs are increased, variable costs decrease rapidly in a low uncertainty environment and change very little in a high uncertainty environment. To put it in another way, high fixed cost systems are very sensitive to the change of uncertainty level while low fixed cost systems are not."

An immediate consequence of this remark is that increasingly higher fixed cost systems become increasingly vulnerable to uncertainty. Therefore one may expect the existence of an upper bound limiting the rise of the fixed cost as a result of such vulnerability. In the context of the technological development of a civilization, one might propose that this remark leads to the same conclusion that was suggested by the anthropologist Tainter [2], that societies evolve towards increasing levels of complexity to solve the problems that they encounter, and inevitably collapse when the complexity reaches an impenetrable upper bound. The question that I would like to pose here is this: is it reasonable to accept such a conclusion as it regards the evolution of a civilization as a system?

I think that the underlying problem here is that we need to have a clear understanding of what we mean by *complexity*. A technological civilization evolves in terms of complexity either by discovering new ways of deploying their current repertoire of discovered universal physical principles, or by discovering new universal physical principles through the creative power of the human mind. The first type of complexity can be quantified in terms of a continuous variable such as the fixed cost of the deployment of technological capability. The second type of complexity is unquantifiable in terms of any type of continuous variable, as it manifests as a sudden discrete transformation, analogous to a phase transition. Therefore, the corollary of the statement by Chen and Galbraith quoted above, as it pertains to civilization as a system, is correct under the assumption that we do not allow the discovery of new universal physical principles. However, if we allow a delicately balanced joint evolution of both categories of complexity, then we no longer have an upper bound to the growth of complexity. The reason is that the effect of the discovery of

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new universal physical principles is to decrease the level of uncertainty faster than the increase of complexity increases the vulnerability to uncertainty fluctuations.

To visualize this process geometrically, imagine a thermodynamic curve representing the power of the system as a function of the fixed cost invested into the system. For the case of a civilization, one can characterize power in terms of Larouche's notion of *relative potential population density* [3]. As we increase the fixed cost, the power of the system increases up to a point. Thereafter, as Chen and Galbraith [1] predict, the law of diminishing returns comes in and the increase slows, and reverts. At this point the difference between a *creative system*, such as our civilization, and a non-creative system becomes critical. In a civilization, the discovery of new universal physical principles essentially allows a discrete shift from one thermodynamic curve to a more favorable one, where the diminishing returns law is delayed thus allowing one to climb higher in power.

Each shift corresponds with an increase in our available resource space as well as an increase in the *energy flux density* used to process our resources. This correlation between increasing energy flux density and increasing relative potential population density was alreaded noted previously by Larouche [3]. At sufficiently high energy flux densities, it becomes possible to recycle resources: for example, water with nuclear desalination, and eventually minerals through fusion-powered element transmutation [4]. By expending the necessary resources to cultivate the creative powers of the human mind among the population, via education as well as various other types of investment, the economy, as a system, acquires the potential for achieving the necessary upshifts in a timely way.

It is also important to realize that a minimum amount of investment is needed to prevent a downshift, as it is necessary, at a minimum, to maintain the current infrastructure as well as transmit current knowledge to future generations. A sudden downshift to lower thermodynamic curves, caused possibly by neglect due to self-destructive policies, may become irreversible if the resources used to climb that curve are no longer available. Over the long run, it will be necessary to expand beyond the Earth, into our Solar System, and into the rest of our galaxy to outrun the limited lifetime of Earth as a life-supporting planet. Thus, if we refuse to upshift to higher curves, we are condemned to eventual but certain extinction as our planet becomes uninhabitable over the next 100-1000 million years, as a result of gradually increasing solar output [5].

The broader philosophical question is: do we live in a universe structured so that subsequent universal physical principles, needed to allow our civilization to expand beyond the Earth, the solar system, and the galaxy, exist and are discoverable? In other words, is the universe structured as is necessary to allow the existence of a sequence of future discoveries of universal physical principles that can allow our civilization to outrun limits to growth indefinitely? The answer depends on the resolution of a deeper underlying question: is the existence of life in our universe an unintended accident or an intended feature of the universe built into the structure of the universe itself from the quantum level and above?

II. ON THE BLACK-SCHOLES MODEL

The mathematical model proposed in the Chen and Galbraith paper [1] is certainly very interesting. In fact, it is the well-known Black-Scholes model. The new insight of Chen and Galbraith [1] is that the same model is a plausible approach for gaining insights on a broader range of problems extending to biophysics and sociology. The Black-Scholes model however is vulnerable to the following criticism:

From the assumption that the evolution of the variable cost S is a lognormal process we get the stochastic differential equation

$$\frac{1}{S}\frac{dS}{dt} = r + \sigma\varepsilon(t), \tag{1}$$

where $\varepsilon(t)$ is a random variable such that

$$\varepsilon(0) = 0, \tag{2}$$
$$\varepsilon(t_1) - \varepsilon(t_2) \sim \mathcal{N}(0, t_1 - t_2),\ \text{for } 0 \leq t_2 < t_1. \tag{3}$$

Here r is the discount rate and σ is the rate of uncertainty. The vulnerability is in the assumption that the increments of $\varepsilon(t)$ are gaussian random variables. For example, it is relatively well-known that in physical systems that deviate from thermodynamic equilibrium you encounter processes that may be approximately gaussian but not precisely gaussian. A classic example is the case of the velocity field of a fluid undergoing a state of hydrodynamic turbulence. It is known, in this case, that the velocity field is not precisely gaussian, and furthermore that the energy cascade process that causes the existence of hydrodynamic turbulence would not be possible if the velocity field were precisely gaussian [6]. In other words, the small deviation from precise gaussian statistics is crucial in enabling a qualitative shift in the dynamics of the fluid into the turbulent state. The essential insight is that the assumption that $\varepsilon(t)$ is gaussian in increments implies the absense of any structure whatsoever in the noise term $\sigma\varepsilon(t)$ at any timescale. Therefore, the question that arises is: to what degree are the qualitative consequences of the Chen–Galbraith theory sensitive to a small violation of this assumption of lognormality?

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