An Analytical Theory of Major factors in Economics

An Extension from the Option Theory

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Abstract

We present an analytical theory of main factors in economic activities. The main result is an formula of variable cost as an analytical function of value of product, fixed cost, lifespan of the investment, discount rate and uncertainty. The formula takes the same form as the Black-Scholes formula of European call options. With this formula, together with the size of output, we can calculate the value or return of an investment systematically. We can also assess how the change of the environmental factors, such as discount rate or uncertainty affect the value of an investment. This theory retains the simplicity and robustness of the Black Scholes option theory. At the same time, it broadens the scope from a financial derivative theory to a general economic theory. It can be applied to understand a broad range of economic and biological systems.

Keywords: Option, fixed cost, variable cost, life span, discount rate, uncertainty

All the calculations, graphs and tables in this paper are contained in an Excel file, which can be downloaded at

http://web.unbc.ca/~chenj/papers/AnalyticalTheory.xlsx

1. Introduction

Black-Scholes formula provides a simple, robust relation among major factors in option pricing. Is it possible to develop a simple, robust relation about major factors in economic activities?

The pioneers of the derivative theories had anticipated new theories with very broad scope. Fischer Black once said,

> I like the beauty and symmetry in Mr. Treynor's equilibrium models so much that I started designing them myself. I worked on models in several areas:
>
> Monetary theory
> Business cycles
> Options and warrants
>
> For 20 years, I have been struggling to show people the beauty in these models to pass on knowledge I received from Mr. Treynor.
>
> In monetary theory --- the theory of how money is related to economic activity --- I am still struggling. In business cycle theory --- the theory of fluctuation in the economy --- I am still struggling. In options and warrants, though, people see the beauty. (Mehrling, 2005, p. 93)

The above statement shows that Black thought that financial instruments and real economic activities can be modeled with very similar methods.

Soon after the development of the option theory, many works have been done to apply the theory to project investments. This type of works is often grouped under the name of the real option theory. The concept of real option has become an important part of the theory of investment decisions. It has been included in many corporate finance textbooks. The methodologies involved in the works of the real option theory are very diverse and often very complicated. Is it possible to develop a simple theory of project investment that is general enough to capture the relation among major factors in economic activities?

Investment under Uncertainty (Dixit and Pindyck, 1994) is the standard reference book on the real option theory. In a footnote in the book, they made an intriguing observation about Feynman's path integral method:

> In quantum electrodynamics the result proves to have immense practical utility. In fact, it underlies Feynman's (1949) diagrammatic technique for summing probabilities over all possible paths of a particle. His approach, developed before dynamic programming and Ito's Lemma had been thought of, was an amazing achievement. Since the dependent variable --- the probability amplitude --- in quantum electrodynamics is complex valued, the analogy with dynamic programing and contingent claims valuation may not extend beyond the mathematical formalism. If it does, then in addition to all his achievements in physics, Feynman could be claimed as the father of financial economics. (Dixit and Pindyck, 1994, p. 123)

The method of Feynman path integral is not restricted to complex valued functions. The analogy does extend to economic activities. By applying Feynman-Kac formula, an extension of Feynman's method, we obtain a differential equation that is almost identical to the Black-Scholes equation. The Black-Scholes equation has a negative sign in front of the time derivative, while our equation has a positive sign in front of the time derivative. From the physics perspective, our equation represents a thermodynamic process while the Black-Scholes equation represents a reverse thermodynamic process. From the economic perspective, Black-Scholes equation solves the current price of derivative securities when the future payout is determined while our equation solves the expected variable cost in the future when the current fixed investment is determined. These two theories solve different types of problems in economic activities. Then we set up the initial condition so that the investment will breakeven as the duration of the investment approaches zero. This is similar to Black-Scholes theory, where the end value of an option is equal to its payoff. We solve the equation to obtain an analytical formula of the variable cost as a function of output value, fixed cost, duration of investment, discount rate and uncertainty (Chen, 2005, 2006a). The formula takes the same form as the Black-Scholes formula for the European call options. However, the meanings of the parameters in this theory differ from that in the option theory. Together with the quantity of the output, we can calculate the return of the investment in a simple and systematic way. We will discuss some of the properties of this theory.

First, variable cost is a decreasing function of fixed cost. In examples on project investment in textbooks, it is often assumed that higher fixed cost investment has lower variable cost. This theory provides an analytical formulation for this common sense. The tradeoff between fixed cost and variable cost is universal in economic activities. Shops located near high traffic flows generate high sales volume per unit time. But the rental costs in such locations are also higher. Well trained employees work more efficiently. But employee training is costly. People with higher education levels on average command higher income. But education takes time, effort and money. Highways with more lanes experience less traffic congestion. But it is more expensive to build wider roads.

Second, variable cost is an increasing function of duration of investment. The following comments from Jack Treynor about Black-Scholes theory provides a relevant background to understand the importance of duration of investment:

> Time has always been a pesky problem for economists, who have dealt with it by
>
> 1. Restricting their model to perpetuities (Modigliani and Miller).
> 2. Focusing on one-period problems (Markowitz's portfolio balancing model).
> 3. Reducing the dynamic flow of economic events to a static long run and a static short run (Alfred Marshall).
>
> That these pioneers in quantifying the previous unquantifiable ducked the problems is a measure of what Black-Scholes accomplished (Treynor, 1996).

Our theory extends the Black-Scholes methodology from financial instruments to real economic activities. By considering corporate finance problems in a finite time horizon, we are able to provide a more precise understanding of the problems related to capital structure considered by

Modigliani and Miller (Chen, 2006b). By working in a continuous time framework instead of a one period framework, we are able to obtain a more refined understanding of relations among risk, discounting and duration of projects (Chen, 2012). By identifying long run cost as fixed cost and short run cost as variable cost and establishing their relations, we provide an analytical theory of economic dynamics that was conceived by Alfred Marshall qualitatively (Chen, 2005).

Third, variable cost is an increasing function of uncertainty. (In this work, we use the words, risk and uncertainty, interchangeably.) In traditional literature, the impact of risk is represented by the discounting rate. Cash flows with higher risk are discounted with higher rate, as determined by the CAPM theory. However, cash flows themselves are affected by the level of uncertainty as operating costs are directly affected by the level of uncertainty. This theory models the relation between uncertainty and cost directly.

Fourth, decision making under different kinds of environment can be easily understood. People and organizations attempt to maximize wealth level in any environment. From the calculation of this theory, when uncertainty level is low, investment with high fixed cost and long duration will generate high economic value; when uncertainty level is high, investment with low fixed cost and short duration will break even quicker. This means in stable environment, organisms and organizations tend to invest heavily for long term. However, this type of investment will suffer heavily when unforeseeable shock occurs. Hence, "stability is destabilizing".

Fifth, this theory can capture subtler relations among major factors in economic activities and in life. In the last several decades, behavioral economics has been very popular. Two of the well-known anomalies about human psychology are hyperbolic discounting and magnitude effect (Thaler,1981; Ainslie, 1992). From the calculation of this theory, low discount rate environment stimulate high fixed cost, long-term investment. So low discount rate is intrinsically related to high fixed cost (magnitude effect) and long duration (hyperbolic discounting). Many of the psychological "anomalies" recorded in literature are really wisdoms obtained from long time evolutionary processes.

An analytical theory makes it easy to experiment with ideas and insights. This is especially helpful in teaching. Through experience, people learn that in a project, the proper amount of fixed investment depend on many factors, such as market size of the product, discount rate, uncertainty and project duration. To enhance the value of a project, it is essential to consider all these factors together in decision making. However, without an analytical theory, it is very difficult to make a precise estimate how each factor affects the value of a project. Furthermore, the insight gained from many years of experience cannot be passed onto others easily. This analytical theory contains a formula of variable cost as a mathematical function of other factors. With this formula, students can experiment with an Excel sheet to see how the change of each factor affects the return of a project. Since all the results are calculated from simple analytical formulas, they can be reproduced and applied easily by researchers and students. All calculations, graphics and tables in this paper are contained in an Excel file, which can be downloaded.

2. A mathematical theory of production

The theory described in this section can be applied to both biological and economic systems. For simplicity of exposition, we will use the language of economics. However the extension to biological system is straight forward.

We start the investigation by asking: What are the most fundamental properties of organisms and organizations? How do we represent these fundamental properties in a mathematical theory? First, organisms and organizations need to obtain resources from the environment to compensate for the continuous diffusion of resources required to maintain various functions. This fundamental property can be represented mathematically by lognormal processes, which contain both a growth term and a dissipation term.

Suppose S represents the amount of resources accumulated by an organism or the unit price of a commodity, r, the rate of resource extraction or the expected rate of change of price and σ, the rate of diffusion of resources or the rate of volatility of price change. Then the process of S can be represented by the lognormal process

$$\frac{dS}{S} = rdt + \sigma dz \,. \tag{1}$$

where

$dz = \varepsilon\sqrt{dt}, \quad \varepsilon \in N(0,1)$ is a random variable with standard Gaussian distribution

The process (1) is a stochastic process. Although a stochastic process will generate many different outcomes over time, we are mostly interested in the average outcomes from such processes. For example, although the movement of individual gas molecules is very volatile, air in a room, which consists of many gas molecules, generates a stable pressure and temperature. We usually study the average outcomes of stochastic processes by looking at the averages of the underlying stochastic variables and their functions. These investigations often transform stochastic processes into their corresponding deterministic equations. For example, heat is a random movement of molecules. Yet the heat process is often studied by using heat equations, a type of deterministic partial differential equation.

Feynman (1948) developed a method of averaging stochastic processes under very general conditions, which is usually called path integral. Kac (1951) extended Feynman's method into a mapping between stochastic processes and partial differential equations, which was later known as the Feynman-Kac formula. According to the Feynman-Kac formula (Øksendal, 1998, p. 135), if

$$C(t,S) = e^{-qt} E(f(S_t)) \tag{2}$$

is the expected value of a function of S at time t discounted at the rate q, then $C(t,S)$ satisfies the following equation

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - qC \tag{3}$$

with

$$C(0,S) = f(S) \tag{4}$$

It should be noted that many functions of S satisfy equation (3). The specific property of a particular function is determined by the initial condition (4). This is similar to the Black-Scholes option theory. The Black-Scholes equation is satisfied by any derivative securities. It is the end condition at contract maturity that determines the specific property of a particular derivative security.

Second, for an organism or an organization to be viable, the total cost of extracting resources has to be less than the gain from the amount of resources extracted, or the total cost of operation has to be less than the total revenue. Costs include fixed cost and variable cost. In general, production factors that last for a long time, such as capital equipment, are considered fixed cost while production factors that last for a short time, such as raw materials, are considered variable costs. If employees are on long term contracts, they may be better understood as fixed costs, although in the economic literature, they are usually classified as variable costs. Typically, a lower variable cost system requires a larger investment in fixed costs, though the converse is not necessarily true. Organisms and organizations can adjust their level of fixed and variable costs to achieve high level of return on their investment. Intuitively, in a large and stable market, firms will invest heavily in fixed cost to reduce variable cost, thus achieving a higher level of economy of scale. In a small or volatile market, firms will invest less in fixed cost to maintain a high level of flexibility. In the following, we will examine the relation between fixed cost and variable cost in a very simple project.

Suppose there is a project with a duration that is infinitesimally small. It only has enough time to produce one unit of product. If the fixed cost is lower than the value of the product, in order to avoid arbitrage opportunity, the variable cost should be the difference between the value of the product and the fixed cost. If the fixed cost is higher than the value of this product, there should be no extra variable cost needed for the product. Mathematically, the relation between fixed cost, variable cost and the value of product in this case is the following:

$$C = \max(S - K, 0) \tag{5}$$

Where S is the value of the product, C is the variable cost and K is the fixed cost of the project. When the duration of a project is of a finite value T, relation (5) can be extended into

$$C(0,S) = \max(S - K, 0) \tag{6}$$

as the initial condition for equation (3). Equation (3) with initial condition (6) can be solved to obtain

$$C = Se^{(r-q)T} N(d_1) - Ke^{-qT} N(d_2) \tag{7}$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a standardized normal random variable. From (6), the solution of the equation (3) can be interpreted as the variable cost of the project. However, we will investigate shortly whether the function represented in formula (7) has common properties of variable costs. For a given investment problem, different parties may select different discount rates. To simplify our investigation, we will make the discount rate equal to the expected rate of growth. This is to set

$$q = r \tag{8}$$

This choice of discount rate can be understood from two perspectives. From a biological perspective, fast growing organisms also have a high probability of death. In a steady state, the growth rate has to be equal to the death rate. In the biological literature, the discount rate is usually set equal to the growth rate (Stearns, 1992). From the perspective of economics, in option theory, the discount rate is set equal to the risk free rate. The level of risk of an option contract is represented by implied volatility, which does not necessarily equate with past volatility or future expected volatility. Some people do not agree with the economic logic behind the mathematical derivation of Black-Scholes equation that made the risk related discount rate disappear (Treynor, 1996). However, the disappearance of the separate discount rate greatly simplified our understanding of how option values are related to market variables. From both a biological and economic perspective, this choice of discount rate provides a good starting point for further investigation.

With q equals to r, equation (3) becomes

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{9}$$

and solution (7) becomes

$$C = SN(d_1) - Ke^{-rT} N(d_2) \tag{10}$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Formula (10) provides an analytical formula of C, variable cost as a function of S, product value, K, fixed cost, σ, uncertainty, T, duration of project and r, discount rate of a firm. Similar to understanding in physics, the calculated variable cost is the average expected cost of variable inputs. With an analytical formula, we can calculate how variable cost changes with respect to other major factors in economic and biological activities. Formula (10) takes the same form as the Black-Scholes formula for European call options. But the meanings of the parameters in this theory differ from that in the option theory.

We will briefly examine the properties of formula (10) as a representation of variable cost. First, the variable cost is always less than the value of the product when the fixed cost is positive. No one will invest in a project if the expected variable cost is higher than the product value. Second, when the fixed cost is zero, the expected variable cost is equal to the value of the product. When the fixed cost approaches zero, the expected variable cost will approach the value of the product. This means that businesses need to make a fixed investment before they can expect a profit. Similarly, all organisms need to invest in a fixed structure before they can extract resources profitably. Some do not agree with this statement and provide examples of low fixed cost investment with high profits, such as J. K. Rowling writing Harry Potter books. Our results are about the statistical average. While a small percentage of authors earn high incomes from blockbusters, an average author does not earn a high income. Third, when fixed costs, K, are higher, variable costs, C, are lower. Fourth, for the same amount of the fixed cost, when the duration of a project, T, is longer, the variable cost is higher. This shows that investment value depreciates with time. Fifth, when risk, σ, increases, the variable cost increases. Sixth, when the discount rate becomes lower, the variable cost decreases. This is due to the lower cost of borrowing. All these properties are consistent with our intuitive understanding of and empirical patterns in production processes.

After obtaining the formula for the variable cost in production, we can calculate the expected profit and rate of return of an investment. Suppose the volume of output during the project life is Q, which is bound by production capacity or market size. During the project life, we assume the present value of the product to be S and the variable cost to be C. Then the total present value of the product and the total cost of production are

$$SQ \text{ and } CQ + K \tag{11}$$

respectively. The net present value of the project is

$$QS - (QC + K) = Q(S - C) - K \tag{12}$$

The rate of return of this project can be represented by

$$\frac{QS - (K + QC)}{K + QC} = \frac{QS}{K + QC} - 1 \tag{13}$$

It is often convenient to represent S as the value of output from a project over one unit of time. If the project lasts for T units of time, the net present value of the project is

$$TS - (TC + K) = T(S - C) - K \tag{14}$$

The rate of return of this project can be represented by

$$\frac{TS - (K + TC)}{K + TC} = \frac{TS}{K + TC} - 1 \tag{15}$$

Unlike a conceptual framework, this mathematical theory enables us to make quantitative calculations of returns of different projects under different kinds of environments.

Since our theory is very similar to Black-Scholes option theory, it is essential to compare the basic equations of Black-Scholes theory and our theory. The Black-Scholes equation is

$$-\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{16}$$

The basic equation in our theory is

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{9}$$

The Black-Scholes equation has a negative sign in front of the time derivative. From the physics perspective, our equation represents a thermodynamic process while the Black-Scholes equation

represents a reverse thermodynamic process. From the economic perspective, Black-Scholes equation solves the current price of derivative securities when the future payout is determined while our equation solves the expected variable cost in the future when the current fixed cost is determined. These two theories solve different problems in economic activities.

In the next several sections, we will provide more detailed analysis on the relation of various factors and returns.

3. Basic properties of the theory

This theory provides quantitative measurements about how major factors in economic and biological systems affect costs and returns of the systems. In the following, we will present some of the calculations

Fixed cost and uncertainty:

By calculating variable costs from (10), we find that, as fixed costs are increased, variable costs decrease rapidly in a low uncertainty environment and change very little in a high uncertainty environment. To put it in another way, high fixed cost systems are very sensitive to the change of uncertainty level while low fixed cost systems are not. This is illustrated in Figure 1.

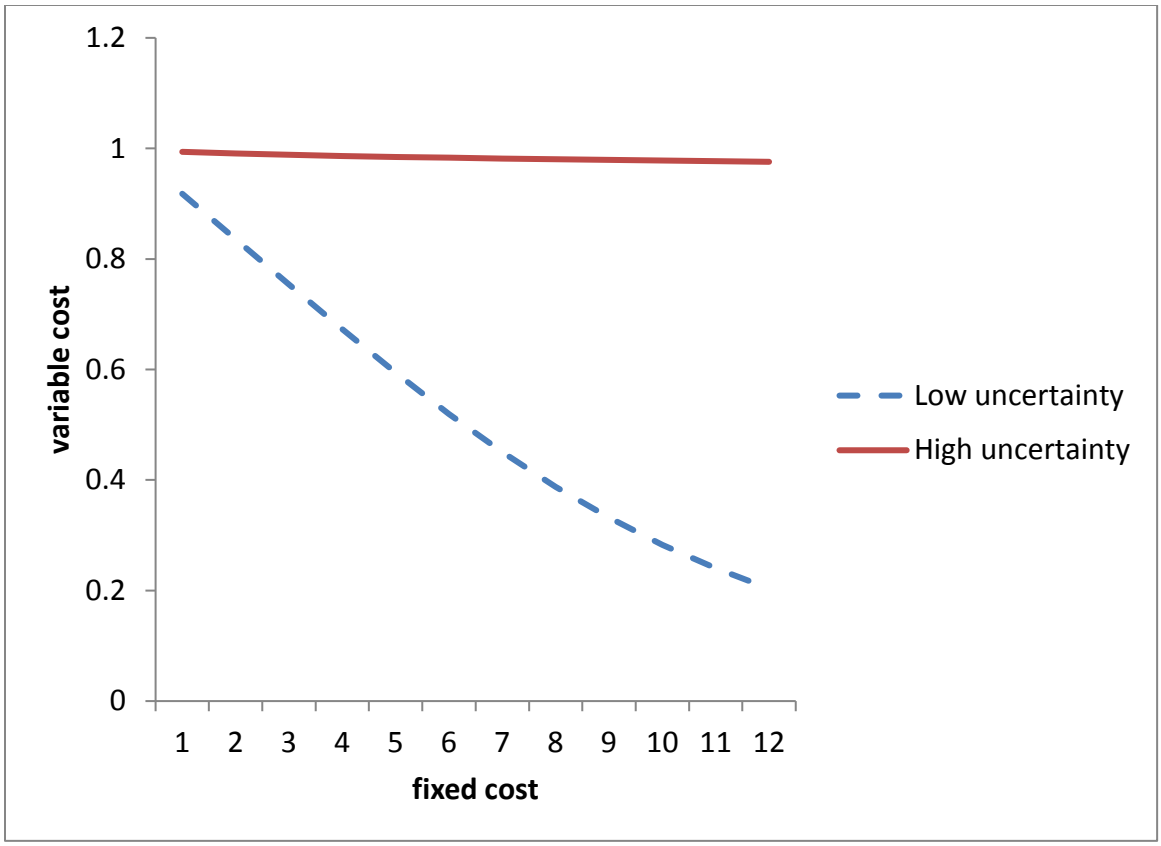

Figure 1. Fixed cost and uncertainty: In a low uncertainty environment, variable cost drops sharply as fixed costs are increased. In a high uncertainty environment, variable costs change little with the level of fixed cost.

The above calculation indicates that systems with higher fixed investment are more effective in a low uncertainty environment and systems with lower fixed investment are more flexible in high uncertainty environment. Mature industries, such as household supplies, are dominated by established large companies such as Proctor &Gamble while innovative industries, such as IT, are pioneered by small and new firms. Microsoft, Apple, Yahoo, Google, Facebook and countless other innovative businesses are started by one or two individuals and not by established firms. Despite the financial and technical clout of large firms, small firms account for a disproportionately high share of innovative activity (Acs and Audretsch, 1990). Similarly, in scientific research, mature areas are generally dominated by top researchers from elite schools, while scientific revolutions are often initiated by newcomers or outsiders (Kuhn, 1996).

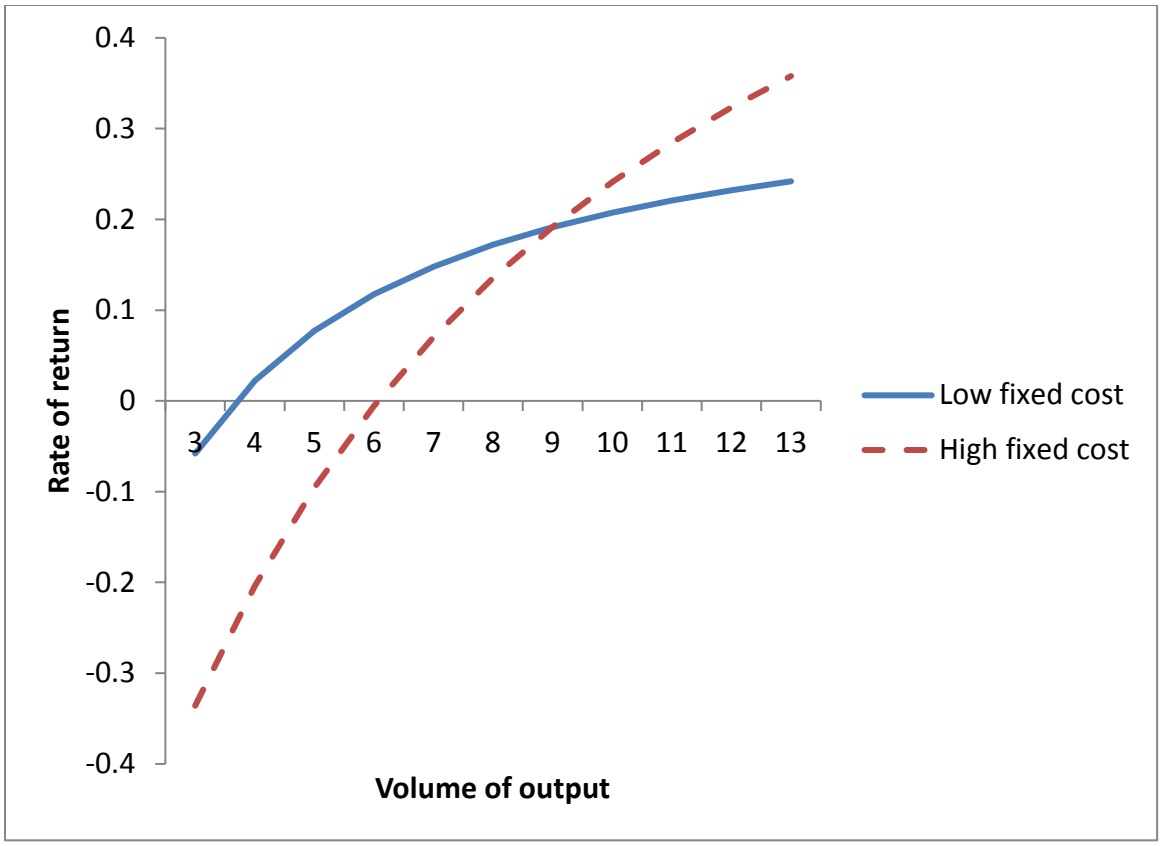

Figure 2. Fixed cost and the volume of output: For a high fixed cost investment, the breakeven market size is higher and the return curve is steeper. The opposite is true for a low fixed cost investment.

Fixed cost and the volume of output or market size:

We now discuss the returns of investment on projects of different fixed costs with respect to the volume of output or market size. Figure 2 is the graphic representation of (14) for different levels of fixed costs. In general, higher fixed cost projects need higher output volume to breakeven. At the same time, higher fixed cost projects, which have lower variable costs in production, earn higher rates of return in large markets.

We can see from the above discussion that the proper level of fixed investment in a project depends on the expectation of the level of uncertainty and the size of the market. When the outlook is stable and the market size is large, projects with high fixed investment earn higher rates of return. When the outlook is uncertain or market size is small, projects with low fixed cost break even more quickly.

In the ecological system, the market size can be understood as the size of resource base. When resources are abundant, an ecological system can support large, complex organisms (Colinvaux, 1978). Physicists and biologists are often puzzled by the apparent tendency for biological systems to form complex structures, which seems to contradict the second law of thermodynamics (Schneider and Sagan, 2005; Rubí, 2008). However, once we realize that systems of higher fixed cost provide higher return in the resource rich and stable environments, this evolutionary pattern becomes easy to understand. An example from physiology will highlight the tradeoff between fixed and variable cost with different levels of output.

> An increased oxygen capacity of the blood, caused by the presence of a respiratory pigment, reduces the volume of blood that must be pumped to supply oxygen to the tissues. …The higher the oxygen capacity of the blood, the less volume needs to be pumped. There is a trade-off here between the cost of providing the respiratory pigment and the cost of pumping, and the question is, Which strategy pays best? It seems that for highly active animals a high oxygen capacity is most important; for slow and sluggish animals it may be more economical to avoid a heavy investment in the synthesis of high concentrations of a respiratory pigment. (Schmidt-Nielson, 1997, p. 120)

This is another example of fixed cost, variable cost trade-off. For high output systems (highly active animals) investment is fixed cost (respiratory pigment) is favored while for low output systems (slow and sluggish animals) high variable cost (more pumping) is preferred. Pumping is variable cost compared with respiratory pigment because respiratory pigment lasts much longer.

With the volatile commodity market, people become aware of the problem of resource depletion. Many people have advocated the increase of efficiency as a way of reduce energy consumption. Will the increase of efficiency reduce overall resource consumption? Jevons made the following observation more than one hundred years ago.

> It is credibly stated, too, that a manufacturer often spends no more in fuel where it is dear than where it is cheap. But persons will commit a great oversight here if they overlook the cost of improved and complicated engine, is higher than that of a simple one. The question is one of capital against current expenditure. … It is wholly a confusion of ideas to suppose that the economic use of fuel is equivalent to the diminished consumption. The very contrary is the truth. As a rule, new modes of economy will lead to an increase of consumption according to a principle recognized in many parallel instances. (Jevons, 1865 (1965), p. xxxv and p. 140)

Put it in another way, the improvement of technology is to achieve lower variable cost at the expense of higher fixed cost. Since it takes larger output for higher fixed cost systems to breakeven, to earn a positive return for higher fixed cost systems, the total use of energy has to be higher than before. That is, technology advancement in energy efficiency will increase the total energy consumption. Jevons' statement has stood the test of time. Indeed, the total consumption of energy has kept growing, almost uninterrupted decades after decades, in the last several centuries, along

with the continuous efficiency gain of the energy conversion (Inhaber, 1997; Smil, 2003; Hall, 2004).

Take hybrid cars as an example. Hybrid cars have two engines, one internal combustion engine, like conventional cars and one electric engine. This adds to the manufacturing cost (and hence resource consumption) of hybrid cars. If the owner of a hybrid car drives very little, the total resource consumption from a hybrid car is actually higher than a conventional car. Only when a hybrid car is used extensively, it becomes less wasteful relative to a conventional car. Therefore, the use of a hybrid car, when manufacturing cost is included, guarantees high resource consumption.

Fixed cost and return

We will examine how different level of fixed cost investment will affect the value of a project. From Figure 3, as the fixed cost of a project is increased, the net present value of the project will increase initially. When the level of fixed cost is at certain level, its further increase will reduce the value of the project. Eventually the value of the project will become negative if its fixed cost is too high. Education is a type of fixed cost in our life. We generally regard education a worthwhile investment. But the majority of people will not pursue Ph.D. degrees.

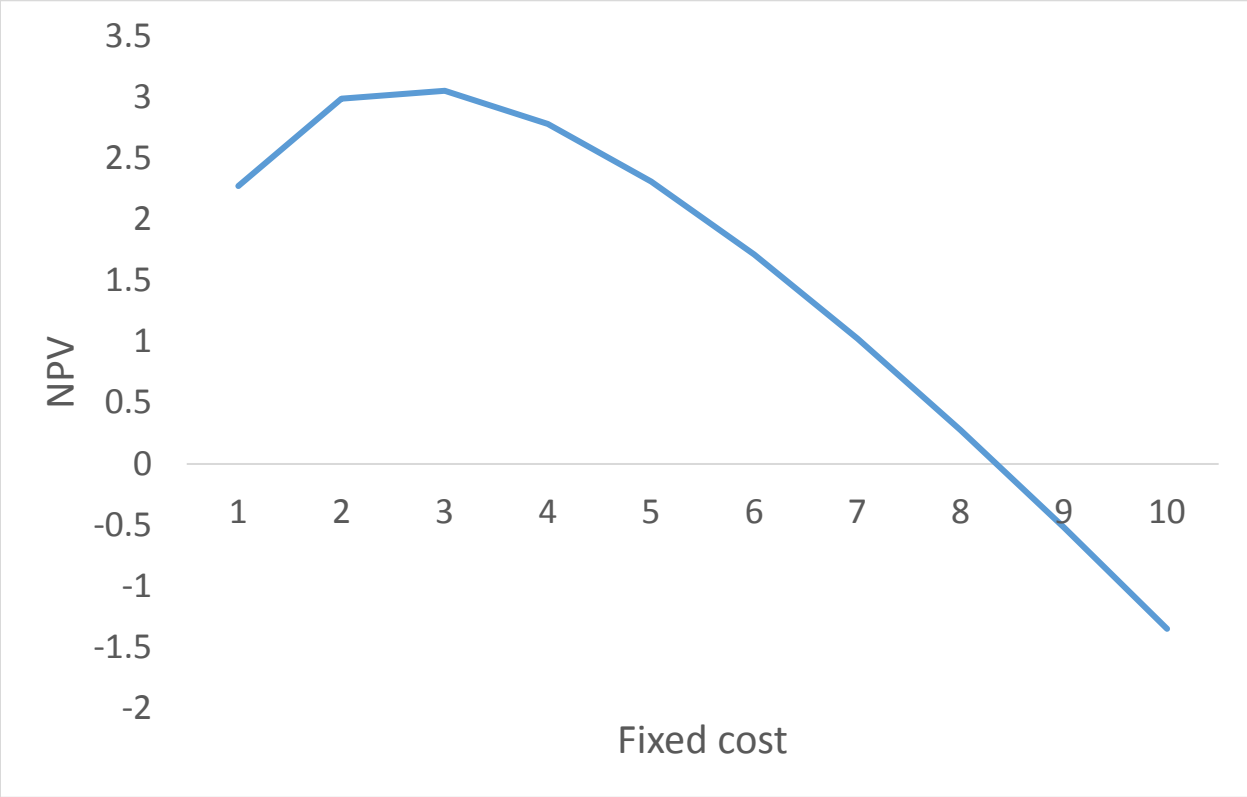

Figure 3. Fixed cost and return:

Duration of the project and return

We will study how the level of the duration of projects affects its return. If the duration of a project is too short, we may not be able to recoup the fixed cost invested in the project. If the duration of a project is too long, the variable cost, or the maintenance cost may become too high. With the mathematical theory, we can make quantitative calculations. The detailed calculation is illustrated in Figure 4. Its shape is very similar to Figure 3. From Figure 4, as the lifespan of a project is increased, the net present value of the project will increase initially. When the lifespan is at certain level, its further increase will reduce the value of the project. Eventually the value of the project will become negative if its lifespan is too high. It explains why individual life does not go on forever. Instead, it is of higher return for animals to have finite life span and produce offspring. This also explains why most businesses fail in the end (Ormerod, 2005).

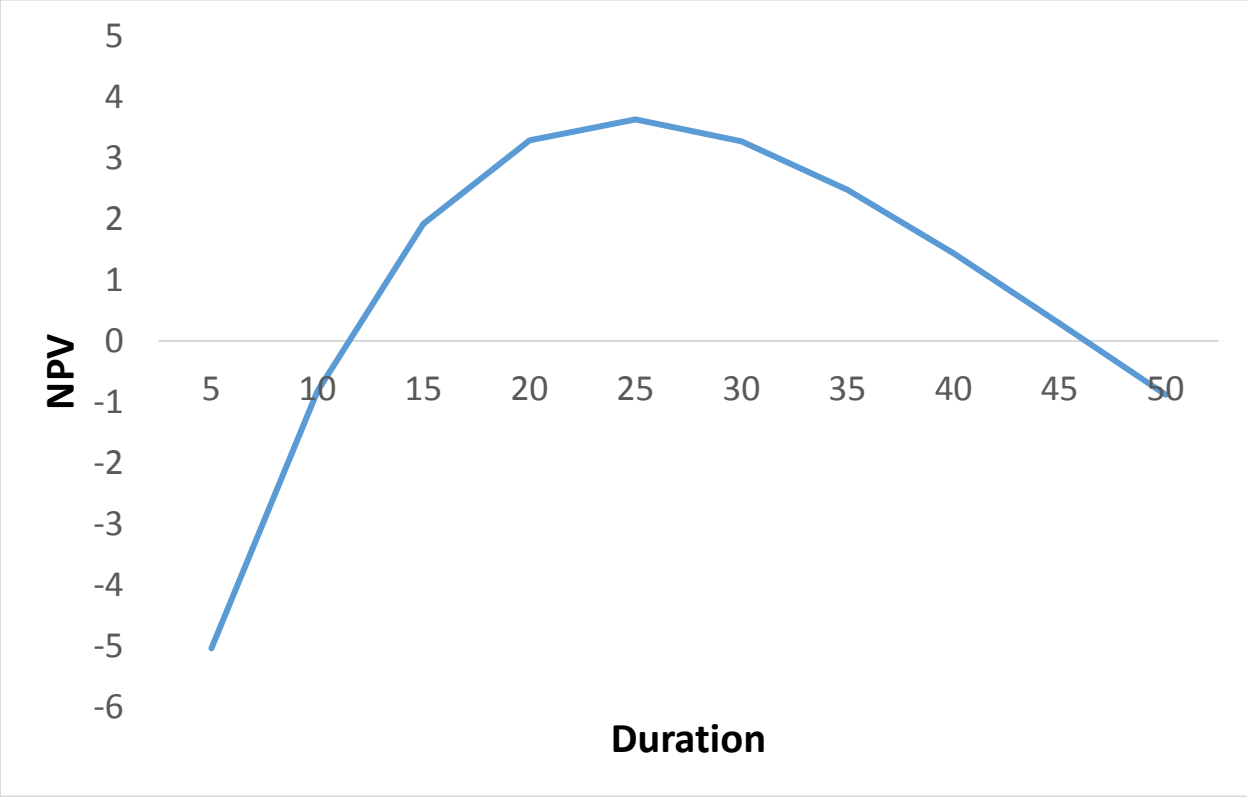

Figure 4. Duration and return

Calculation also shows that when the level of fixed cost increases, the length of duration for a project to earn a positive return also increases. This suggests that large animals and large projects, which have higher fixed cost, often have longer life. There is an empirical regularity that animals of larger sizes generally live longer (Whitfield, 2006). The relation between fixed cost and duration can be also applied to human relation. In child bearing, women spend much more effort than men.

Therefore we would expect women value long term relation while men often seek short term relation, which is indeed the case most of the time (Pinker, 1997).

Since higher fixed cost systems have longer life span than lower fixed cost systems, the mutation rates of lower fixed cost systems are faster. This gives lower fixed cost systems advantages in adapting changes. For example, AIDS virus is much smaller than human beings and can mutate much faster. This makes it difficult for humans to develop natural immune response or develop drugs to fight against AIDS virus. However, higher animals develop a general strategy in immunes systems that has been very effective most of the time. Instead of developing one kind of antibody, our immune systems produce millions of different types of antibodies. It is highly likely that for any kind of bacteria or viruses, there is a suitable antibody to destroy them. This strategy is very effective but very expensive, because our body needs to produce many different kinds of antibodies that are useless most of the time. When we are too young, too old or too weak, our bodies don't have enough energy to produce large amount of antibodies. That is when we get sick often.

From calculation, when the duration of a project keep increasing, the return of a project will eventually turn negative. Hence, duration of a project or an organism cannot become infinite. For life to continue, there has to be a systematic ways to generate new organisms from old organisms. From earlier calculation, for a system to have a positive return, fixed assets have to be invested first. Thus old generations have to transfer part of their resources to younger generations as the seed capital before younger generations can maintain positive return. Therefore, there is a universal necessity of resource transfer from the old generation to the younger generation in biological world. "Higher" animals, such as mammals, generally provide more investment to each child than "lower" animals, such as fish. In human societies, parents provide their children for some years before they become financial independent. In general, wealthy societies provide more investment to children before they start to compete in the market than poor societies. In businesses, new projects are heavily subsidized at their beginning stages by cash flows from profitable mature projects.

Since project life or organism life cannot last forever, resource transfer from organism to organism or from project to project is unavoidable. However, the process of transfer is often the source of many conflicts. Businesses prefer lower tax rates. Educational institutions prefer higher government revenues. Each child wants more attention from parents. Parents would like to distribute resources more or less evenly among different children. Mature industries, which need little R&D expense, prefer low tax systems. High tech industries, which rely heavily on universities to provide new technologies, employees and users, strongly advocate government support in new technologies. In good times, financial institutions preach the virtue of free market to pursue high profits. In bad times, the same institutions will remind the public how government support can ensure financial stability of the nation. The amount of resource transfer and the method of resource transfer often define the characteristics of a species or a society.

Fixed cost and discount rate:

We discuss how the level of fixed cost affects the preference for discount rates. We will calculate how variable costs change with different discount rates. When discount rates are decreased, the variable costs of high fixed cost systems decrease faster than the variable costs of low fixed cost systems (Figure 5). This indicates that high fixed cost systems have more incentive to maintain low discount rates or lending rates. This result helps us understand why prevailing lending rates are different at different areas or times.

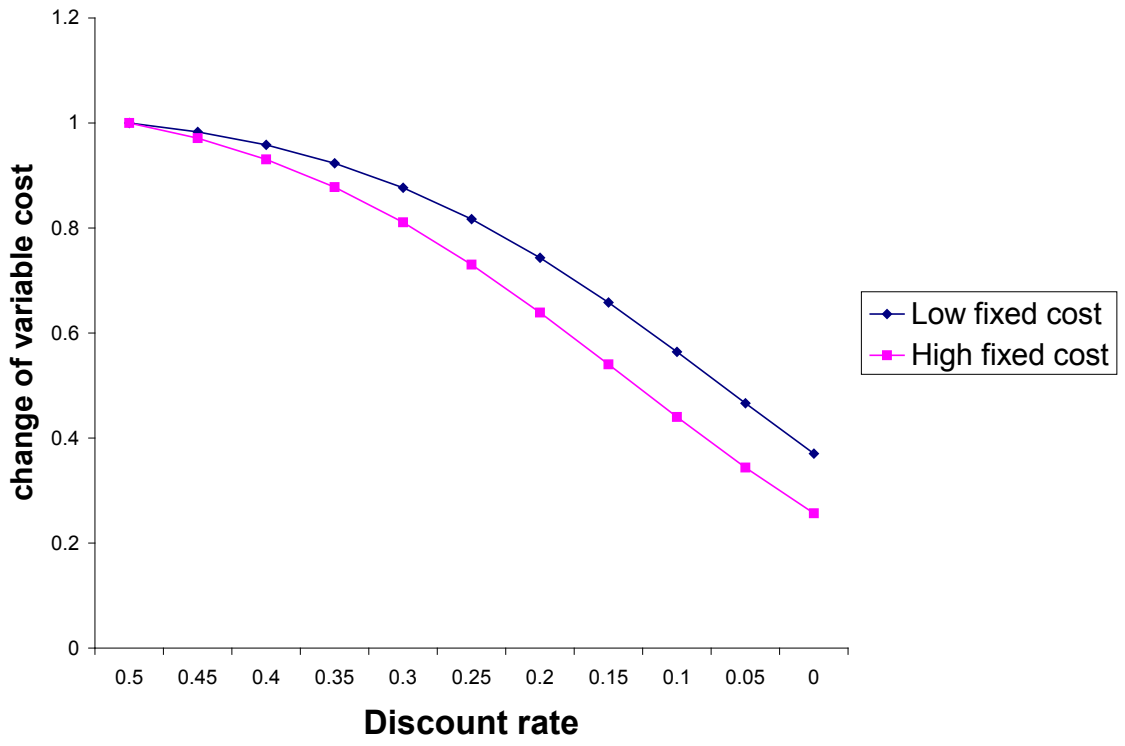

Figure 5. Fixed cost and discount rate: When discount rates are decreased, variable costs of high fixed cost systems decreases faster than variable costs of low fixed cost systems.

In poor countries, lending rates are very high; in wealthy countries, lending rates charged by regular financial institutions, other than unsecured personal loans, such as credit card debts, are generally very low. To maintain a low level of lending rates, many credit and legal agencies are needed to inform and enforce, which is very costly. As wealthy countries are of high fixed cost, they are willing to put up the high cost of credit and legal agencies because the efficiency gain from lower lending rate is higher in high fixed cost systems. In the last several hundred years, there is in general an upward trend in living standard worldwide. There is also a downward trend in interest rates (Newell and Pizer, 2003).

Empirical investigations show that the human mind intuitively understands the relation between discount rate and different level of assets. In the field of human psychology, there is an empirical regularity called the "magnitude effect" (small outcomes are discounted more than large ones). Most studies that vary outcome size have found that large outcomes are discounted at a lower rate than small ones. In Thaler's (1981) study, respondents were, on average, indifferent between $15 immediately and $60 in a year, $250 immediately and $350 in a year, and $3000 immediately and $4,000 in a year, implying discount rates of 139%, 34% and 29%, respectively. Since the human mind is an adaptation to the needs of survival and reproduction, evaluating the relation between discount rate and amount of investment must be a common task in our evolutionary past.

Differences in fixed costs in child bearing between women and men also affect the differences in discount rates between them. Women spend much more effort in child bearing. From our theory, the high fixed investment women put in child bearing would make women's discount rate lower than men's. An informal survey conducted in a class showed that discount rates of the female students are lower than that of the male students.

Our understanding about discount rate and fixed cost is similar to an earlier work by Ainslie and Herrnstein (1981):

> The biological value of a low discount rate is limited by its requiring the organism to detect which one of all the events occurring over a preceding period of hours or days led to a particular reinforcer. As the discounting rate falls, the informational load increases. Without substantial discounting, a reinforcer would act with nearly full force not only on the behaviors that immediately preceded it, but also on those that had been emitted in past hours or days. The task of factoring out which behaviors had actually led to reward could exceed the information processing capacity of a species.

Discount rate and project duration:

When the discount rate becomes lower, the variable cost of a project will decrease and profit will increase. Projects with different lengths of duration will be affected differently from the reduction of discount rates. Figure 6 presents the ratios of profits between projects at low and high discount rates at different levels of project duration. As project lengths are increased, the ratios mostly increase as well. This indicates that projects with longer duration benefit more from the reduction of interest rates. Keynes made a similar argument that as interest increases, the optimal duration of production process shortened (Keynes, 1936, p. 216).

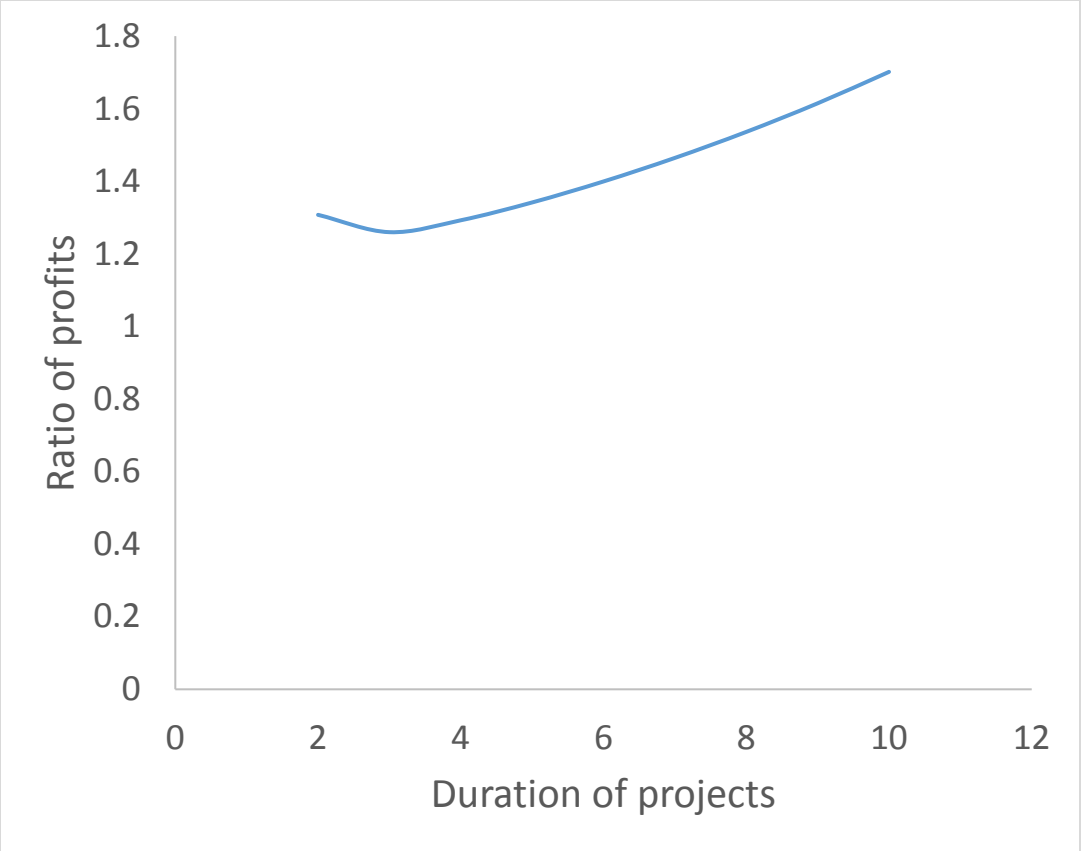

Figure 6. Project duration and discount rate: the ratios of profits between projects at low and high discount rates at different levels of project duration

Next we calculate the breakeven point of a project with respect to the project duration and the discount rate. Let us assume that project output per unit of time is one. The calculation from formula (14) shows that it requires lower discount rate to breakeven when the project duration is lengthened. The calculation results are illustrated in Figure 7. Many empirical studies have documented that humans, as well as other animals, often discount long duration events at lower rates than short duration events (Frederick, Loewenstein and O'Donoghue, 2004). This pattern is called hyperbolic discounting. The calculation provides a possible explanation for hyperbolic discounting. Since it takes lower discount rates for long duration projects to breakeven, human and other animals minds will discount long duration projects with lower rates.

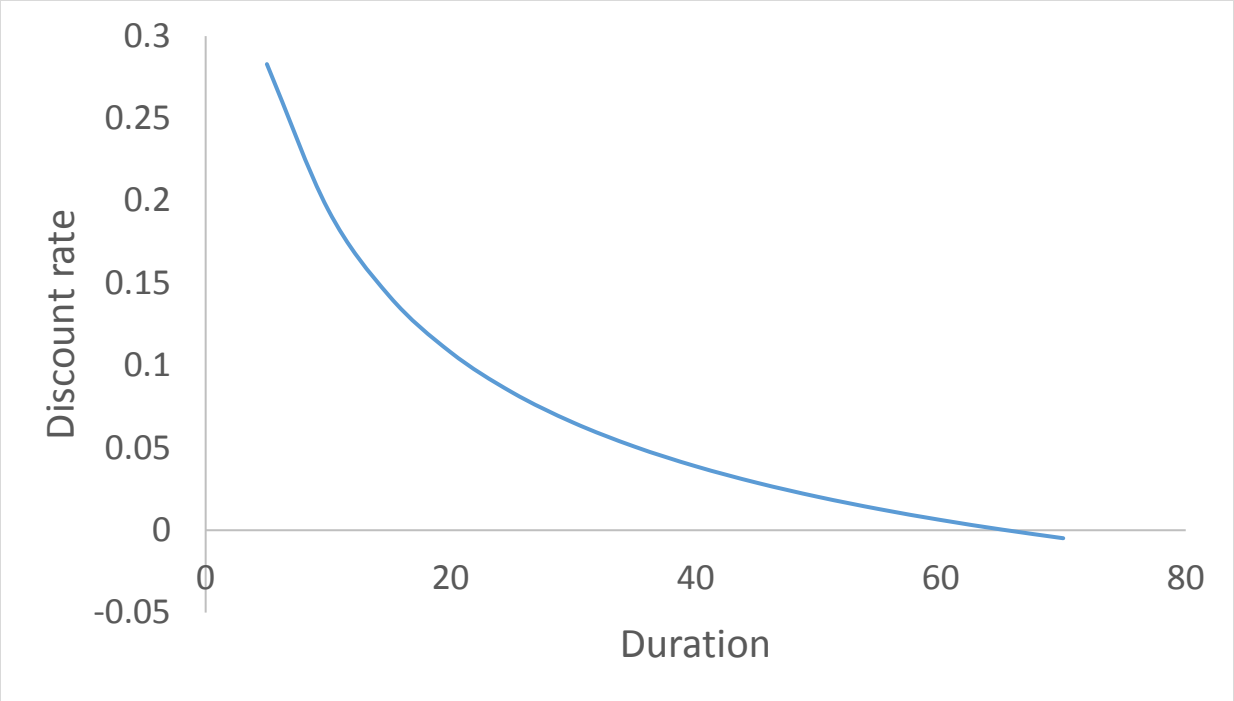

Figure 7. Required discount rate for the project to break even at different project duration: As project duration increases, required discount rate for the project to break even decreased. This provides a possible explanation for hyperbolic discounting.

In the following, we will present more empirical evidences about the inverse relationship between discount rate and duration of project or span of life. Fecundity, as well as mortality rate, is proximity for discount rate (Stearns, 1992). Lane (2002) provided a detailed discussion about the tradeoff between longevity and fecundity in the biological systems.

> Notwithstanding difficulties in specifying the maximum lifespan and reproductive potential of animals in the wild, or even in zoos, the answer is an unequivocal yes. With a few exceptions, usually explicable by particular circumstances, there is indeed a strong inverse relationship between fecundity and maximum lifespan. Mice, for example, start breeding at about six weeks old, produce many litters a year, and live for about three years. Domestic cats start breeding at about one year, produce two or three litters annually, and live for about 15 to 20 years. Herbivores usually have one offspring a year and live for 30 to 40 years. The implication is that high fecundity has a cost in terms of survival, and conversely, that investing in long-term survival reduces fecundity.

> Do factors that increase lifespan decrease fecundity? There are number of indications that they do. Calorie restriction, for example, in which animals are fed a balanced low-calorie diet, usually increase maximum life span by 30 to 50 per cent, and lower fecundity during the period of dietary restriction. … The rationale in the wild seems clear enough: if food is scarce, unrestrained breeding would threaten the lives of parents as well as offspring. Calorie restriction simulates mild starvation and increase stress-resistance in general. Animals that survive the famine are restored to normal fecundity in times of plenty. But then, if the evolved response to famine is to put life on hold until times of plenty, we would expect to find an inverse relationship between fecundity and survival. (Lane, 2002, p. 229)

Lane went on to provide many more examples on the inverse relation between longevity and fecundity.

In human society, we often use longevity, or duration of human life as an indicator of the quality of a social environment. At the same time, societies that enjoy a long life span, such as Japan, are often concerned about below replacement fertility. Intuitively, the aging population needs a great amount of resources to maintain their health, which reduces the amount of resources available to support children. Hence, there is a natural tradeoff between longevity and fertility. This result has important policy implications on the balance between resource distribution on longevity and fertility. In a society with below replacement fertility, this poses a great challenge to maintain a sustainable society.

Discount rate and uncertainty

Variable cost is an increasing function of discount rate. When uncertainty is low, variable cost is much lower with a low level of discount rate. When uncertainty is high, variable costs are not very sensitive to discount rate. Therefore, it is often more effective to reduce discount rate in a stable environment. Figure 8 presents the change of variable costs at different levels of discount rates when levels of uncertainty are low and high. It shows that the reduction of the variable cost is more significant at a low uncertainty level. This explains why *r* species, which have high discount rates, often thrive in highly uncertain environments. It may show why low interest rates, in a climate of economic crisis, have little effect on the level of perceived profitability and therefore on activity. This is called "pushing on a string."

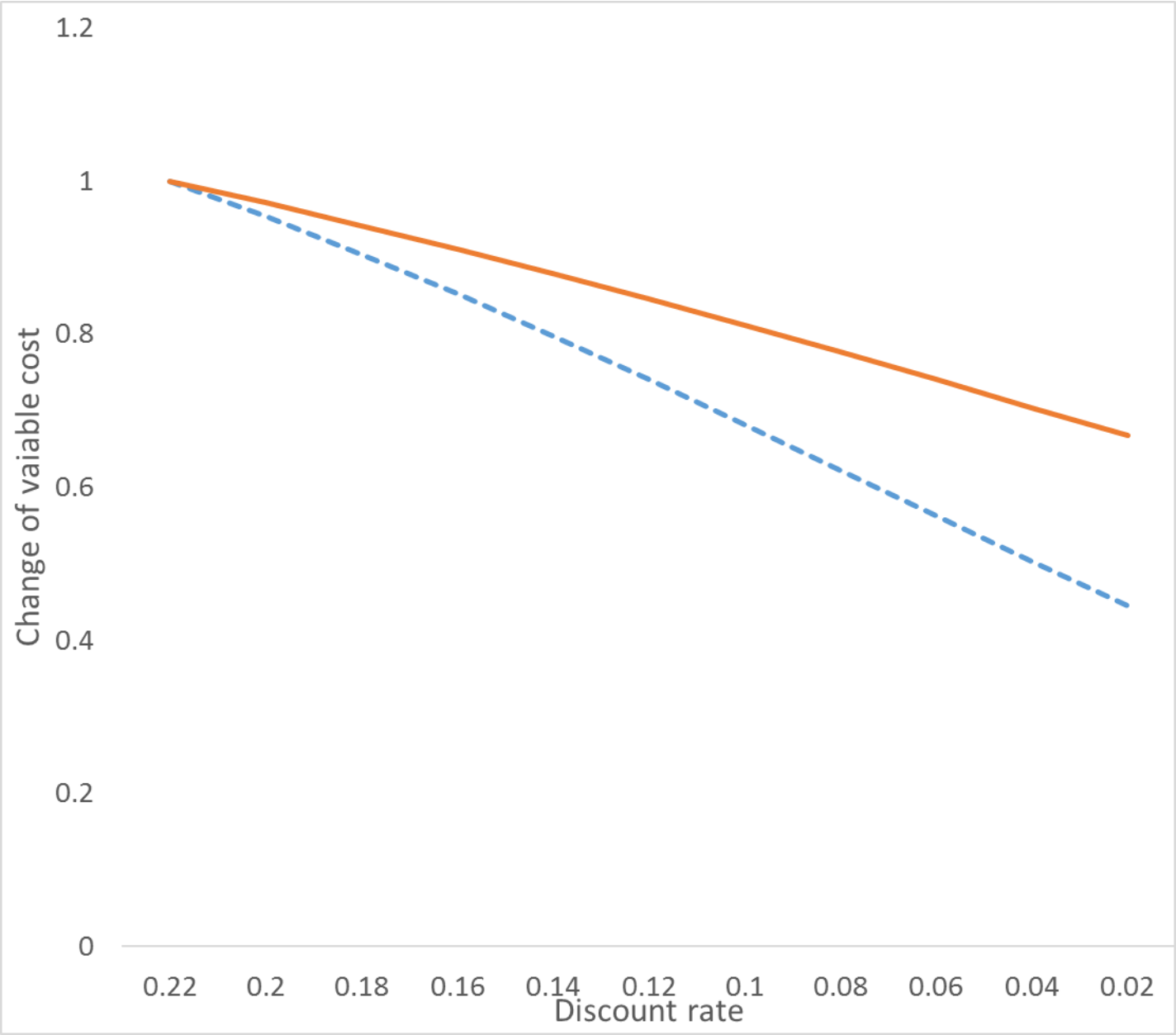

Figure 8.

The same idea about the relationship between discount rate and uncertainty had been reached earlier. "The same discount curve that is optimally steep for an organism's intelligence in a poorly

predictable environment will make him unnecessarily shortsighted in a more predictable environment (Ainslie, 1992, p. 86).

4. Decision making in specific environment

Decision makers will attempt to maximize the value or return of an investment in any given environment. Very often, the levels of discount rate and uncertainty are external environmental factors not controlled by businesses. People may choose the level of fixed cost and lifespan of projects to maximize the value or return of a project in that specific environment. For brevity, we will assume decision makers maximize value of a project in the following discussion.

Environment of different discount rates

Let discount rate take two different values at 3% and 10% per annum respectively and keep the level of uncertainty same. We will choose the level of fixed cost and lifespan of the project to maximize formula (14), the net present value of the project. The left side of Table 1 lists the maximization result.

discount rate	0.03	0.1		0.03	0.1
annual output	1	1		1	1
Fixed cost	7.1	3.9		7.1	3.9
duration of project	35.7	13.6		35.7	13.6
uncertainty	0.3	0.3		0.8	0.8
variable cost	0.46	0.42		0.97	0.86
NPV	12.3	4.0		-6.2	-2.0

Table 1.

From Table 1, we find that when interest rate is lower, the amount of fixed investment is larger, investment duration is longer and the net present value is higher. So investors normally prefer low interest rate environment. However, the net present values are expected returns calculated at the beginning of a project. The actual returns depend on future environment. Suppose after the projects are built, the actual level of uncertainty becomes 80% per annum instead of 30% per annum as previously expected. We can recalculate the net present values from (14) to find the net present

value of the first project, built in the low interest rate environment of 3%, becomes -6.2 billion dollars while the net present value of the second project, built in the high interest rate environment of 10%, becomes -2.0 billion dollars. The right side of the above table lists the calculated results in the new environment. Both projects suffer losses. But the first project suffers much more losses. When environmental conditions change, values of investment in low interest rate environment experience larger fluctuations. In other words, the monetary policy of low interest rate will generate greater business cycles. So business cycles are greatly tied to monetary policies. This theory provides a simple and clear understanding about the level of interest rate and magnitude of business cycles.

Stability is destabilizing

Hyman Minsky once said, "Stability is destabilizing". What does it mean exactly? With an analytical theory, we can obtain a very clear understanding from an example similar to the one at the beginning of this section. Suppose in two countries A and B, annual output is 1 billion dollars. Suppose interest rate is 5% per annum in both countries. Uncertainty rate is 30% per annum in country A and 55% per annum in country B. Decision makers attempt to maximize the net present value of investment project. How much will be the desired fixed costs and how long will the expected project last? What are the net present values of projects in countries A and B?

We attempt to maximize (14), which is net present value of an investment by changing fixed cost, K and duration, T when uncertainty rates are set at 30% and 55% per annum respectively. Left side of Table 2 lists the calculated results.

uncertainty	0.3	0.55		0.8	0.8
annual output	1	1		1	1
Fixed cost	5.8	2.1		5.8	2.1
discount rate	0.05	0.05		0.05	0.05
duration of project	25.3	12.1		25.3	12.1
variable cost	0.44	0.64		0.94	0.82
NPV	8.5	2.3		-4.4	0.0

Table 2.

From Table 2, we find that when uncertainty rate is lower, the amount of fixed investment is larger, investment duration is longer and the net present value is higher. So investors normally prefer low uncertainty rate environment. However, the net present values are only expected returns calculated at the beginning of a project. The actual returns depend on future environment. Suppose after the projects are built, the actual level of uncertainty becomes 80% per annum in both countries due to circumstances unforeseen by decision makers, such as a global financial crisis. We recalculate (14) to find the new net present values. The net present value of the first project, built in the low uncertainty rate environment of 30%, is -4.4 billion dollars while the net present value of the second project, built in the high uncertainty rate environment of 55%, is 0.0 billion dollars. The

calculated results are listed at the right side of Table 2. The first project suffers heavy losses while the second project barely breaks even. When environmental conditions change dramatically, values of investment in formerly stable environment experience large fluctuations. In other words, "Stability is destabilizing".

With this analytical theory, simulation is very simple. It enables us to perceive long term consequences of economic policies and social structures clearly. This theory provides a simple and clear understanding of monetary policies and business cycles. Detailed discussion on monetary policy and business cycles is presented in Chen (2012, 2016). Many economists and policymakers do sense the long term implications of their policies. However, without a simple tool to communicate the long term impacts, most people naturally focus their attention to short term outcomes.

5. Relations among parameters

So far, we have assumed the parameters in the production processes, fixed cost, duration of the project, discount rate, uncertainty and quantity of output, to be independent variables. But in reality, these parameters often have complex and varying relations. It takes detailed knowledge and deep insight about each system to model these relations well. In the following, we will present some simple attempts to model such relations.

All parameters in our theory, except uncertainty, correspond to directly observable quantities. This is very similar to option pricing theory, in which all parameters, except volatility, correspond to directly observable quantities. In option pricing theory, volatility is often called implied volatility because volatility is implied from the option prices. Similarly in our theory, uncertainty is implied from the expected variable costs. Indeed, the value of uncertainty can involve many factors. The meaning of uncertainty rate can be very different in different applications.

Economy of scale and the law of diminishing return

All economic systems experience economy of scale and the law of diminishing return at the same time. This can be modelled by setting uncertainty as a function of output. This can be modeled with uncertainty, σ, as an increasing function of the volume of the output. Specifically, we can assume

$$\sigma = \sigma_0 + lQ$$

Where σ_0 is the base level of uncertainty, Q is the volume of output and $l > 0$ is a coefficient. Intuitively, when the size of a company increases and the business expands, the internal coordination and external marketing becomes more complex. With the new assumption, we can calculate the rate of return of production from formula (13). The result from the calculation is presented in Figure 9. From Figure 9, the rate of return initially increases with the production scale, which is well known as the economy of scale. When the size of the output increases further, the rate of return begin to decline. This is the law of diminishing return. In specific applications, we

can analyze how each factor influence the shape of return curves and try to obtain high rate of returns for our investment.

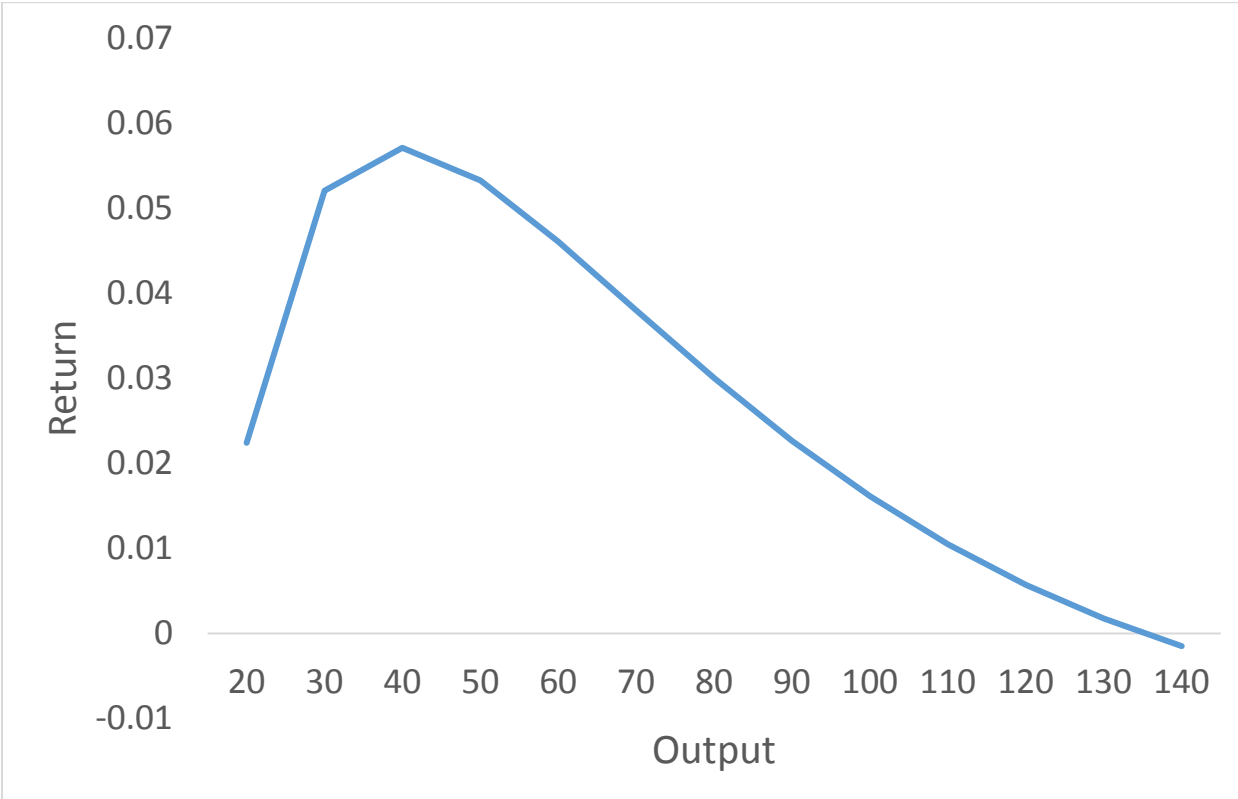

Figure 9. Volume of output and the rate of return: The rate of return of a project with respect to volume of output, when uncertainty is an increasing function of volume of output

Increasing fixed cost to reduce uncertainty

When the fixed cost of a system increases, the increased fixed cost can often help reduce uncertainty. Many organisms, from stegosaurus to turtles, invest in armor to decrease predation. Air conditioning and heating systems can reduce the uncertainty of temperature in a building. But air conditioning requires an increase in electricity consumption. Insurance can reduce uncertainty of large losses for the policyholders. Yet, insurance premium needs to be paid. This pattern can be modeled with uncertainty, σ, as a decreasing function of the fixed cost. Specifically, we can assume

$$\sigma = \sigma_0 + e^{-lK}$$

where σ_0 is the base level of uncertainty, K is the fixed cost and $l > 0$ is a coefficient. Assume the unit product value is 1, discount rate is 5% per annum, duration of project is 10 years. Assume σ_0 is 20% per annum and l is 0.2. Calculated rate of return from the project with different levels of

fixed cost is shown in Figure 10. When the level of fixed cost is increased, the rate of return increases initially and then declines.

Many decision makings involve the spending of fixed cost to reduce uncertainty, such as unemployment insurance, old age insurance, medical insurance, and governments' guarantee to financial institutions. People often take polar positions in debating these issues. A good quantitative model will help us reach compromise among various parties.

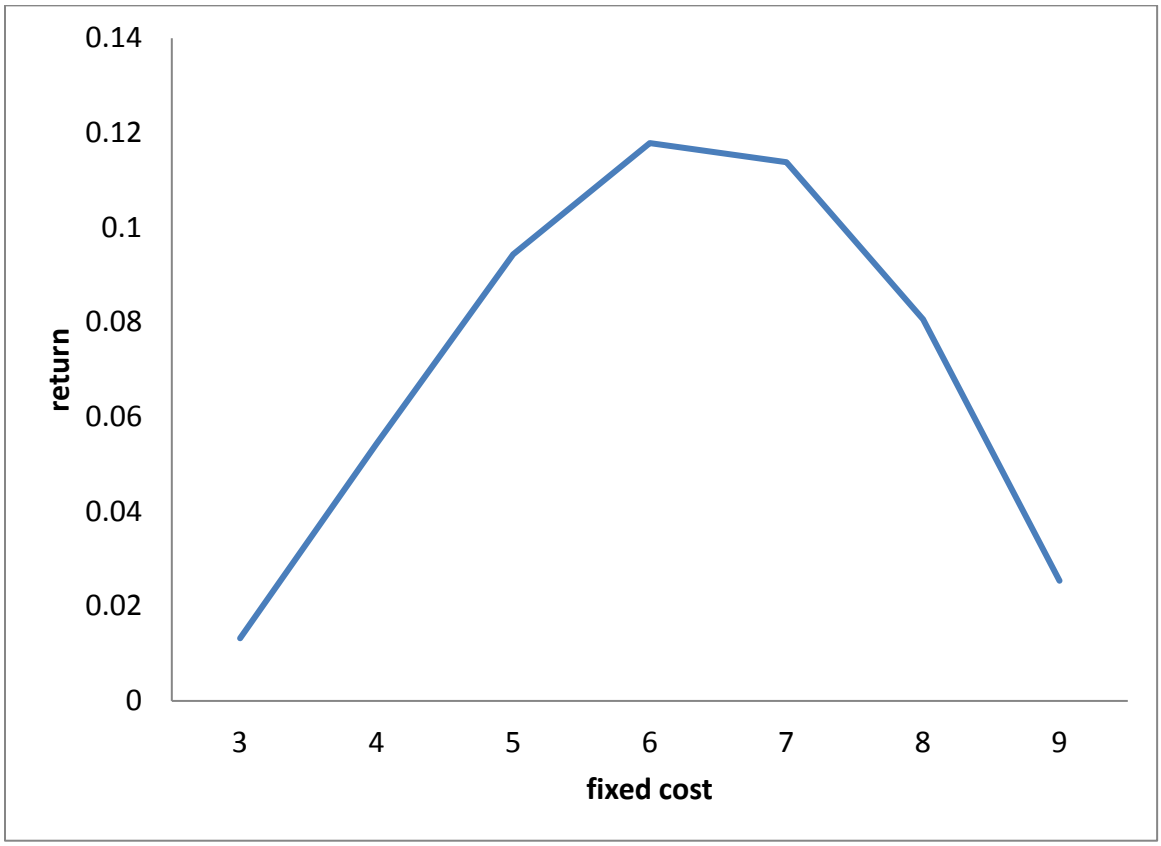

Figure 10. Increasing fixed cost to reduce uncertainty, showing the relation between level of fixed cost investment and rate of return

Resource abundance and investment decisions

One way to represent the resource abundance and quality is by the level of uncertainty rate. In physics, the term representing uncertainty rate in lognormal process is often called the diffusion rate, or the dissipation rate. A higher dissipation rate means that more energy is wasted as heat and less energy is applied to do useful work, indicating low quality of energy fuels. For example, when a dry cell gets discharged, its internal resistance gradually increases and more energy turns into

unusable heat. The quality of the dry cell declines over time. So the quality of resources can be represented by the (inverse of the) uncertainty rate, or dissipation rate.

In the following, we will apply the analytical theory to understand how the cost of processing natural resources affects the structure and size of economic systems. We will model the increase of processing costs for natural resources through the increase of diffusion rate. From the calculation of (10), when the diffusion rate is higher, the variable cost becomes higher. Intuitively, higher diffusion rates mean more effort is needed to process the same amount of resources. Specifically, the level of diffusion will be modelled as:

$$\sigma = \sigma_0 + lQ$$

σ_0 is the base level of diffusion, which corresponds to the lowest cost in production when resource of highest quality is used. Q is the total output of economy and $l > 0$ is a coefficient. The size of l represents the abundance of low cost resources. When a low-cost resource is abundant, l is small. An increase of output will not increase processing costs substantially. When the low-cost resource is scarce, l is large. An increase of output will require high-cost resources, which increases the processing cost substantially.

We will examine how changes in resource abundance affect the structure of and return on economic activity. For simplicity, we will set $S = 1$, $r = 0.1$, $T = 15$ and $\sigma_0 = 0.4$. We will let l take the values of 0.0025, 0.005 and 0.01 to represent different levels of resource abundance. By maximizing formula (13) with respect to the fixed cost and volume of output at different values of l, we obtain the highest possible rate of return from investment projects in different environments. When $l = 0.0025$, projects obtain the highest possible rate of return of 28% when the fixed cost is 9.5 and market size is 56. When $l = 0.005$, projects obtain the highest possible rate of return of 15% when fixed cost is 4.5 and market size is 33. When $l = 0.01$, projects obtain highest possible rate of return of 6% when the fixed cost is 1.7 and market size is 20. Figure 11 displays the rates of return with respect to different sizes of output by three different projects with different fixed costs corresponding to different levels of resource abundance.

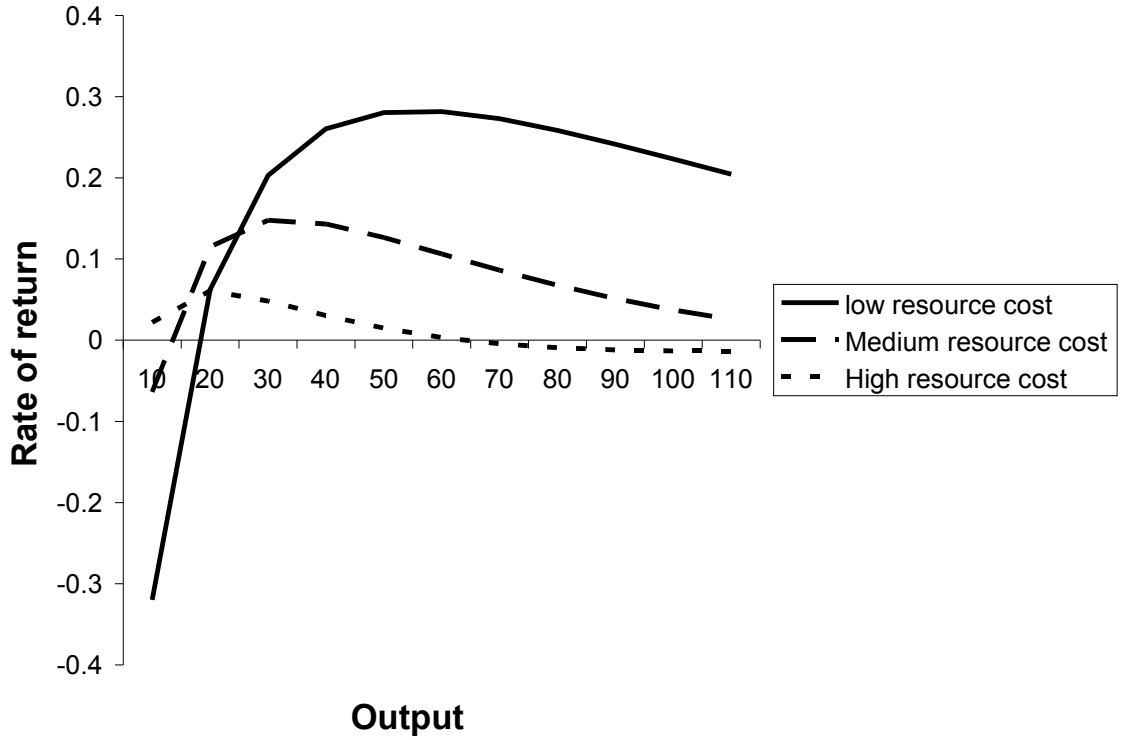

Figure 11. Resource abundance, fixed cost, market size and rates of returns: The rates of return with respect to different output scales for three different projects with different fixed costs corresponding to different levels of resource abundance.

From the above calculation, when resources are abundant and cheap, there is an economic incentive to increase fixed cost and market size to fulfil the possibility of a high rate of return. As a result, the rate of resource consumption is high. When the low-cost resources are gradually depleted, the return from the same high fixed cost system will decline gradually. To understand the precise relation between resource abundance, economic structure and rate of return, we will calculate the rates of return for the same high fixed cost system, which provides the highest possible rate of return, at $l = 0.0025$, when $l = 0.005$ and 0.01. Figure 12 displays the rates of return with respect to different output scales by the same high fixed-cost production system, at different levels of resource abundance. Compared with Figure 11, when the cost of resource production is moderately higher, represented by $l = 0.005$, the difference of return between the existing high–fixed-cost production system and the potential optimal production system is not

large. Furthermore, developing new projects with different levels of fixed cost may require new skills and equipments. Therefore, the incentive to change is small. But when low-cost resources are further depleted, represented by l = 0.01, rates of returns of the high fixed cost production system turn negative at all levels of output, as shown in Figure 12. Reduction of fixed cost and output size are then required to restore economic activities to positive returns.

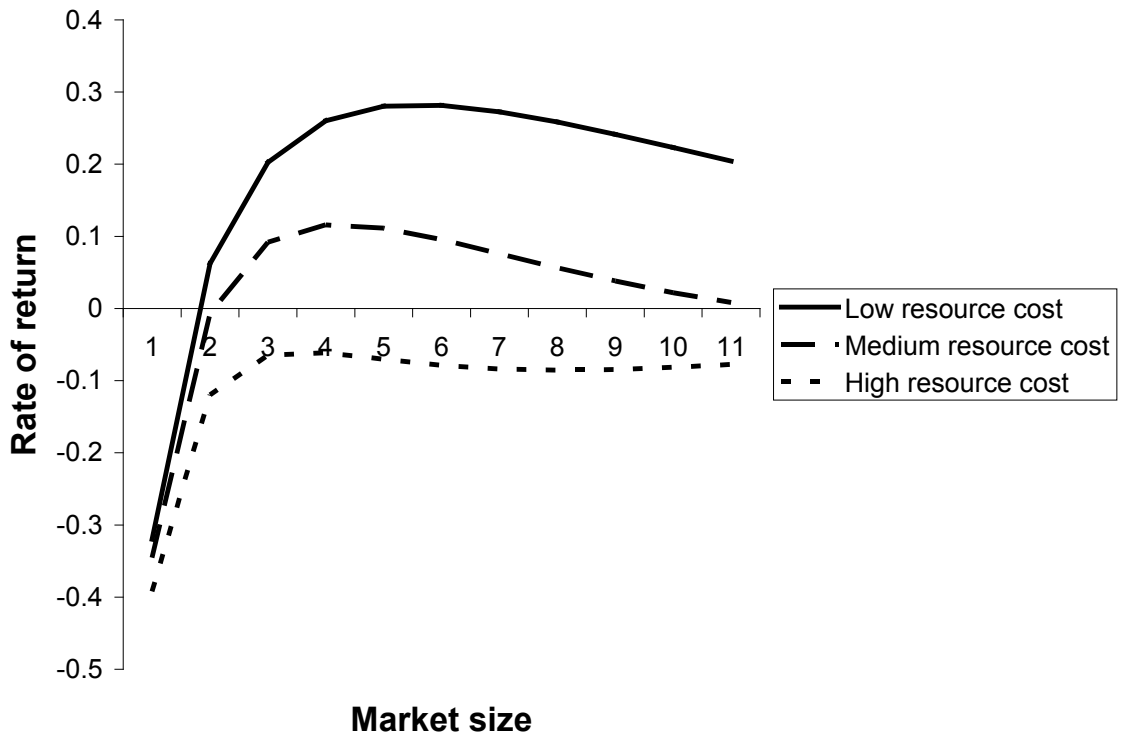

Figure 12. Resource abundance and rates of returns: The rates of return with respect to different output scales for the same high-fixed-cost production system corresponding to different levels of resource abundance.

Since it takes a long time and great effort to adjust institutional structures, it will be very helpful to estimate the state of resource abundance today and in the near future. In a now classic paper titled The End of Cheap Oil, Campbell and Laherrere (1998), after carefully examining the data on oil exploration and production, concluded "What our society does face, and soon, is the end of the abundant and cheap oil on which all industrial nations depend."

6. Concluding Remarks

In this paper, we present an analytical theory of production derived from the biophysical principles. The generality of this theory is a consequence from the generality of physical laws. Physical systems, biological systems and economic systems all follow the same physical laws. This allows us to develop a unified production theory that can be applied to many different fields. Historically, some economic type principles in physics, such as principle of least action and maximum entropy principle (Jaynes, 1957), have been very fruitful in providing unified foundations to very diverse areas of investigation. The production theory presented in this work has provided a unified understanding for a wide range of problems in economics and biology.

Since the theory was developed, it has been applied to project investment, corporate finance, trade and migration, resource and social structures, language and cultures, evolutionary and institutional economics, fiscal and monetary policies, business cycles, firm size and competitions, software development economics and many other problems (Chen, 2016; Chen and Choi, 2009; Chen and Galbraith, 2011, 2012a, 2012b; Liu, Kong, Chen, 2015). There are more detailed relations among these major factors in specific systems. Some relations are explored in this paper. However, many works need to be done to provide more accurate and detailed understanding of the economic and social behaviors.

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