Epilogue

Mathematics, Beauty and Reality: The Evolution of Scientific Theories

When one explores wilderness uninhabited by human beings, it is very difficult to determine where and how to start. It is very difficult to go through forests or bushes. More importantly, it is almost impossible to know if one can get sufficient food supply along the way and what one can get at the end of the journey.

Facing great uncertainty and physical difficulty, early explorers often try to find trails opened and used by animals. These trails greatly expedite the exploration of otherwise impenetrable wilderness. More importantly, there are good reasons for animals to use the trails, which often lead to resource rich destinations.

When one explores intellectual wilderness uninhabited by researchers, it is very difficult to determine where or how to start. More importantly, it is almost impossible to know if one can keep a job for financial survival along the way and what one can get at the end of the journey.

Facing great uncertainty and financial difficulty, early explorers of intellectual wilderness often try to find and use intellectual trails in our mind evolved through our animal ancestors. These intellectual trails greatly expedite the exploration of otherwise infinitely many leads. More importantly, there are good reasons for our animal ancestors to evolve these trails, which often help to obtain resources.

There are many intellectual trails developed in the mind. We will only discuss two of them. The first is mathematics. If some decision making process is truly important and is needed again and again in life, it is highly economical that quantitative modules to be developed in the mind to expedite the process. For example, predators need routinely to assess its distance from the prey, the geometry of the terrain, the speed differential between itself and the prey, the energy cost of chasing down its prey, the probability of success of each chase and the amount of energy it can obtain from prey to determine whether, when and where to start a chase. There are many other sophisticated functions, such as navigating by migrating birds over long distance, that need sophisticated mathematical capabilities. Many animals need to make precise calculation of many of these quantitative problems many times in life. To reduce the cost of estimation, mathematical models must be evolved in their mind so many decision making processes can be simplified into parameter estimation and numerical computation. It is highly likely that, if some function is very important for the survival of the animal, in the process of evolution, this function will become encoded into animal mind. By incurring fixed cost of developing and maintaining such a mathematical function, it reduces the variable cost in each decision making. Therefore, it is natural that a fundamental understanding about life can be expressed as a mathematical theory. More precisely, since entropy is the only mathematical function to measure scarce resources, it is almost inevitable that a basic theory on life and social systems should be a mathematical theory based on entropy.

As we have discussed in this book, low entropy, information, economic value and resources are essentially the same thing looked from different angles. Because of the fundamental importance of entropy in life, human mind, which is an evolutionary product, thinks around entropy. This is why an entropy theory based economic theory turns out to be so simple and universal. This is also why so many people have been engaged in the discussion to apply the concept of entropy in economics for many years, despite the stern reprimand from Paul Samuelson, the most powerful authority in economics:

> And I may add that the sign of a crank or half-baked speculator in the social sciences is his search for something in the social system that corresponds to the physicist's notion of "entropy". (Samuelson, 1972, p. 450)

The second intellectual trail is the sense of beauty. "The geometry of beauty is the visible signal of adaptively valuable objects: safe, food-rich, explorable, learnable habitats, and fertile, healthy dates, mates, and babies." (Pinker, 1997, p. 526) More generally, the sense of beauty is an evolved intuition about resources. Long ago, Eddington noticed the relation between entropy and beauty:

> Suppose that we were asked to arrange the following in two categories – distance, mass, electric force, entropy, beauty, melody. I think there are the strongest grounds for placing entropy alongside beauty and melody, and not with the first three. Entropy is only found when the parts are viewed in association, and it is by viewing or hearing the parts that beauty and melody are discerned. All three are features of arrangement. It is a pregnant thought that one of these three associates should be able to figure as a commonplace quantity of science. The reason why this stranger can pass itself off among the aborigines of the physical world is that it is able to speak their language, viz., the language of arithmetic. It has a measure-number associated with it and so is made quite at home in physics. (Eddington, 1958 (1935), p. 105)

Personally, for many years, I was deeply attracted by the beauty of stochastic processes and their deterministic representations in partial differential equations. In the end, the theory developed in this book was germinated from the theory of stochastic processes and partial differential equations.

The above discussion indicates that beautiful mathematics often has deep connection with the real world. These connections, once established, are often plain and obvious. But the process of establishing the connections may be long and elusive. The understanding about the relation between information and physical entropy provides a good example.

Shortly after Shannon (1948) developed the entropy theory of information, Weaver commented: "Thus when one meets the concept of entropy in communication theory, he has a right to be rather excited --- a right to suspect that one has hold of something that may turn out to be basic and important." (Shannon and Weaver, 1949, p. 13) This sense of excitement attracted a lot of attempts to apply the concept of entropy to many other areas. As it is often the case, earlier attempts to apply some promising intuition do not yield concrete results easily. In an editorial, Shannon tried to discourage the jumping on the bandwagon:

> Workers in other fields should realize that that the basic results of the subject are aimed at a very specific direction, a direction that is not necessarily relevant to such fields as psychology, economics, and other social sciences. Indeed, the hard core of information theory is

> essentially, a branch of mathematics, a strictly deductive system. (Shannon, 1956)

Recent authority reinforces the idea that information theory has only limited connection with physical and social sciences.

> The efforts of physicists to link information theory more closely to statistical physics were less successful. It is true that there are mathematical similarities, and it is true that cross pollination has occurred over the years. However, the problem areas being modeled by these theories are very different, so it is likely that the coupling remains limited.
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> In the early years after 1948, many people, particularly those in the softer sciences, were entranced by the hope of using information theory to bring some mathematical structure into their own fields. In many cases, these people did not realize the extent to which the definition of information was designed to help the communication engineer send messages rather than to help people understand the meaning of messages. In some cases, extreme claims were made about the applicability of information theory, thus embarrassing serious workers in the field. (Gallager, 2001, p. 2694)

If Shannon's entropy theory of information is purely a mathematical theory with little connection with the physical laws, it would be a miracle that information defined as entropy turns out to have some magic technical properties in communication problems. However, once mathematical theories are thought to be a natural part of our evolutionary legacy, it would be natural for entropy theory of information to possess these properties.

> How can the independence of human volition be harmonized with the fact that we are integral parts of a universe which is subject to rigid order of nature's laws? (Planck, 1933, p. 107)

This question is called "one of man's oldest riddles". A major insight from this theory is that human mind, shaped by natural selection, is in tune with natural laws to lower the cost of information processing. Most of information we receive are processed unconsciously. It is only in rare occasions when decision making is needed, information processing becomes conscious. And in most situations, there are no real choices any way. For example, you have the choice to eat or not to eat. But if you decide not to eat, you will be wiped out by natural selection. In a competitive world, one has to follow "optimal" choice, which is not really a choice, on most important decisions to avoid being selected out.

In the following, we will discuss the general patterns of the evolution of scientific theories to understand the origin and process of scientific revolutions.

Scientific theories are developed to reduce the cost of understanding nature, which includes human society. Costs consist of fixed cost and variable cost. Fixed cost helps reduce variable cost. The basic set of fixed assets of a scientific theory is called paradigm (Kuhn, 1996).

> When the individual scientist can take a paradigm for granted, he need no longer, in his major works, attempt to build his field anew, starting from first principles and justifying the use of each concept introduced. (Kuhn, 1996, p. 20)

The establishment of a paradigm, by incurring a common fixed cost, reduces variable cost in scientific development and communication. As a theory matures, its fixed assets accumulated. For a high fixed cost system, its variable cost will be very low when uncertainty is small. This is why science education "is a narrow and rigid education, probably more so than any other except perhaps in orthodox theology" (Kuhn, 1996, p.166).

To further utilize the fixed assets that have been acquired by the scientific community, existing paradigm is being applied to broader and broader fields. "For normal-scientific work, for puzzle-solving within the tradition that the textbooks define, the scientist is almost perfectly equipped. Furthermore, he is well equipped for another task as well --- the generation through normal science of significant crises. When they arise, the scientist is not, of course, equally well prepared. Even though prolonged crises are probably reflected in less rigid educational practice, scientific training is not well designed to produce the man who will easily discover a fresh approach." (Kuhn, 1996, p.166) Therefore, somebody, who "appears with a new candidate for paradigm" is "usually a young man or one new to the field" (Kuhn, 1996, p.166). The following quote about the emission of light or heat has a parallel in scientific research:

> That the whole world is not aglow with radiation is a consequence of a competition between the discarding of energy as radiation and as heat. The products of most reactions are in such intimate contact with their surroundings that any excitation is quickly transferred to the neighboring molecules in the form of thermal motion. However, there are some reactions for which the contact is so weak that the excited state survives long enough for the relatively slow business of squeezing out a photon to occur. (Atkins, 1991, p. 206)

The above quote can be directly translated into a comment about research. That the whole world is not aglow with great idea is a consequence of a competition between the dissemination of information as great idea or as small idea. Most of us have intimate contact with our surroundings that any new idea is quickly transferred to the academic community in the form of thermal motion, or low impact research. However, there are some cases where the contact with the academic community is so weak that ideas hold long enough in the mind for the relatively slow business of squeezing out a truly fundamental theory to occur.

All practicing scientists are educated in a common paradigm, which make it easy for them to communicate with each other. The developers of new paradigms, however, have no such luxury. From information theory, equivocation is high in communication when the receiver of information does not share the same common background or paradigm with the sender of information. The promotion of a fundamentally new idea is in general so difficult that Wallace, the cofounder of the theory of evolution, gave much more credit to the promotion of new ideas over their creation. "No one deserves either praise or blame for the *ideas* that comes to him. … But the *actions* which result from our ideas may properly be so treated, because it is only by patient thought and work that new ideas, if good and true, become adapted and utilized." (George, 1964, p. 280)

Because of the harsh environment to new ideas, many pioneers in scientific research were able to develop and promote new theories only by sheer perseverance. "Neither by poverty, nor by incomprehension of the contemporaries who ruled over the condition of his life and work, did he allow himself to be crippled or discouraged." This is a comment about Kepler's life from Einstein. It is also a reflection of lives of many other pioneers. Their struggle provides a profound testimony that information is costly and information with high value is very costly to obtain.