Chapter 2

The Entropy Theory of Value

2.1. Introduction

Value theory occupies a peculiar position in the development of economic theory. Most of the time, it is a little treaded area to mainstream economists for it is generally thought to be completely resolved. But a major shift in economic thinking often begins with the emergence of new understanding about value. For example, Mill (1871) asserted that he had left nothing in the laws of value for any future economist to clear up, shortly before Jevons and Walras, in the 1870's, developed new theories of value that became the core of neoclassical economics. After neoclassical economics firmly established its dominance, research in the area of value theory became essentially dormant again in the last several decades.

Since all human activities represent extraction and transformation of low entropy from the environment, it is natural to relate economic value to low entropy (Schrodinger, 1944; Prigogine, 1980). Indeed "there have been sporadic suggestions that all economic values can be reduced to a common denominator of low entropy" (Georgescu-Roegen, 1971, p. 283). However, some conceptual difficulties prevented the development of an entropy theory of value.

Georgescu-Roegen thought that linking economic value to low entropy would not be of much help to economists because "he would only be saddled with a new and wholly idle task --- to explain why these coefficients differ from the corresponding price ratios" (Georgescu-Roegen, 1971, p. 283). To this argument we may compare the works of Shannon and Wiener on information theory. Both defined information as the reduction of entropy (Shannon, 1948; Wiener, 1948). However, Shannon further applied the mathematical definition of entropy to obtain some results that are of fundamental importance in information theory, while recognizing that the mathematical definition of information is not identical to the meaning of information common in our daily use. As a result, Shannon established information theory as a science. In this chapter, we show that while economic value is not identical to physical entropy, the entropy theory of value, an analytical theory based on fundamental physical laws, enables us to obtain some results that greatly clarify the meaning of economic value and are highly consistent with our intuitive understanding. Among other things, it offers a clear understanding how institutional structures affect economic value of commodities. Roughly speaking, economic value is the low entropy value of a commodity whose property rights are enforced by governments or other institutions. Since the costs and willingness to enforce property rights on different kinds of commodities are different, the levels of enforcement are different, which, among other factors, causes commodities of similar physical entropy level to be priced very differently.

How is the entropy theory of value related to the existing economic theories of value? Neoclassical economics, the current mainstream economic theory, was developed around 1870 by Jevons, Walras and others. Walras (1954), the chief architect of neoclassical economics, argued that value is a function of scarcity. From the properties that the value of commodities should satisfy, it can be derived that the only mathematical formula to represent value, as a function of scarcity, is the entropy function. This is parallel to the idea that the only mathematical formula to represent information, as a function of probability, is the entropy function (Shannon, 1948). Thus, the entropy theory of value is the analytical formalization of Walras' vision of value as a function of scarcity.

Why both information and economic value are the reduction of entropy? From the entropy law, the most universal law of the nature, the increase of entropy of a system is spontaneous. The reduction of entropy in a system, however, takes effort, which is the base for both information and economic value.

In today's mathematical environment, it is easy to envision an entropy theory of value as the formalization of Walras' theory of value. Historically, however, marginal utility theory of value, which was influenced by Jevons, was easier to define mathematically. Gradually, it becomes the standard economic theory (Fisher, 1925; Debreu, 1959).

While marginal utility is easy to define mathematically, it is difficult to measure empirically (Mirowski, 1989). Indeed the current theory of value does not attempt to measure value empirically. This is reflected in the mathematical tools adopted in the theory: "In the area under discussion it has been essentially a change from calculus to convexity and topological properties, a transformation which has resulted notable gains in the generality and in the simplicity of the theory" (Debreu, 1959, p. x). At the same time, the convexity and topological methods leave no room for a quantitative measure of value. By contrast, the entropy theory of value is established on a measurable mathematical function with clear physical meaning.

Since information is the reduction of entropy, an entropy theory of value is inevitably an information theory of value. The success of Shannon's entropy theory of information stimulated many research efforts in economics (Theil, 1967). However, the information theory of value, or the entropy theory of value, was not developed. Very often, the direction of scientific research is shaped by the thinking of an authority. In an often cited passage, Arrow wrote, "the well-known Shannon measure which has been so useful in communications engineering is not in general appropriate for economic analysis because it gives no weight to the value of the information. If beforehand a large manufacturer regards it as equally likely whether the price of his product will go up or down, then learning which is true conveys no more information, in the Shannon sense, than observing the toss of a fair coin" (Arrow, 1983 (1973), p. 138). The Shannon measure actually carries weight of information. For example, N symbols with identical Shannon measure carry N times more information than a single symbol (Shannon, 1948). Similarly, the value of the information about the future price is higher to a large manufacturer than to a small manufacturer, other things being equal. Later in this chapter, we show that information as an economic commodity shares most of the important properties with physical commodities.

The rest of the chapter is structured as follows. In Section 2.2, we formally develop the mathematical theory of value as entropy. This part extends Shannon's (1948) classic work on information theory. The entropy theory of value provides a quantitative framework to understand how different factors affect the value of a commodity. The influence on value by factors such as scarcity, the number of producers, substitutability and market size of a commodity can be naturally understood from the entropy formula of value. Since scarcity of resources, including human resources, is often regulated by institutional measures such as immigration laws and patent laws, the values of economic commodities are in great part a reflection of institutional structures. In Section 2.3, we utilize the results from information theory, statistical physics and the theory of evolution to discuss the relation between physical entropy value and the economic value. We discuss how this entropy theory of value offers a unifying understanding of the objective and subjective theories of value. In Section 2.4, we discuss how informational and physical commodities share common properties in the light of this entropy theory of value. By resolving the conceptual difficulties that have confounded us for many years, we offer a unified understanding of physical entropy, information and economic value. In Section 2.5, we discuss the relation between economic value and social welfare. Section 2.6 concludes.

2.2. The Properties of an Entropy Theory of Value

Value is a function of scarcity. Scarcity can be defined as a probability measure P in a certain probability space. It is generally agreed that the value of any product satisfies the following properties:

(a) The value of two products should be higher than the value of each of them.

(b) If two products are independent, that is, if the two products are not substitutes or partial substitutes of each other, then the total value of the two products will be the sum of two products.

(c) The value of any product is non-negative.

The only mathematical functions that satisfy all of the above properties are of the form

$$V(P) = -\log_b P \tag{2.1}$$

where b is a positive constant (Applebaum, 1996).

In general, if the scarcity of a service or product, *X*, can be estimated by the probability measure $\{p_1, p_2, \dots p_n\}$, the expected value of this product is the average of the value of each possibility, that is

$$V(X) = \sum_{i=1}^{n} p_i(-\log_b p_i) \qquad (2.2)$$

Therefore, value, just as information, in its general form can be defined as entropy, given that they are the same mathematically. In information theory, the base of the logarithm function is usually chosen to be two because there are two choices of code in information transmission, namely, 0 and 1 (Shannon, 1948). In economics, the base *b* can be understood as the number of producers. In the following we will discuss the properties of value as entropy.

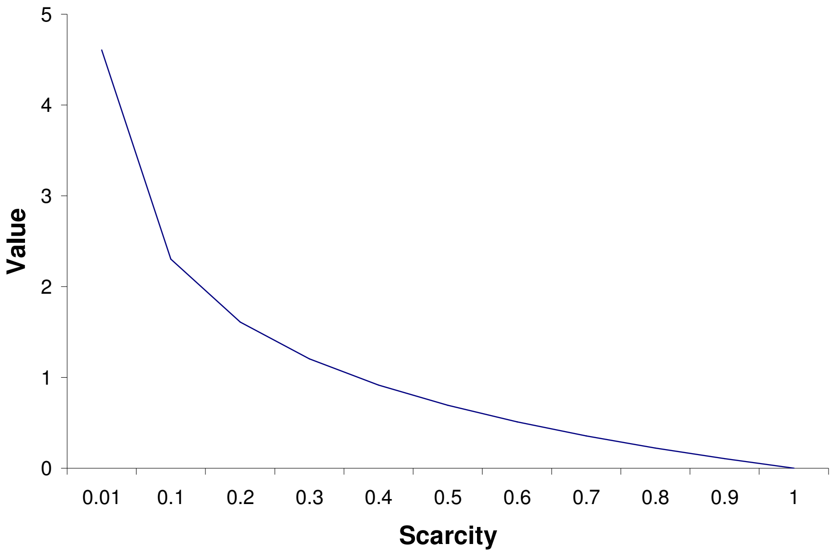

Figure 2.1 Value and scarcity

1. Scarcity and value

Figure 2.1 is a graph of (2.1), which shows that value is an increasing function of scarcity. That is why diamonds are worth more than water. In extreme abundance, i.e., when P=1, -log P =0, the value of a given commodity is equal to zero, even if that commodity is very useful. For example, food is essential for survival. Most countries subsidize food production in various ways to guarantee the abundance of food, which causes its low economic value. This shows that economic value and social value can have divergent valuations.

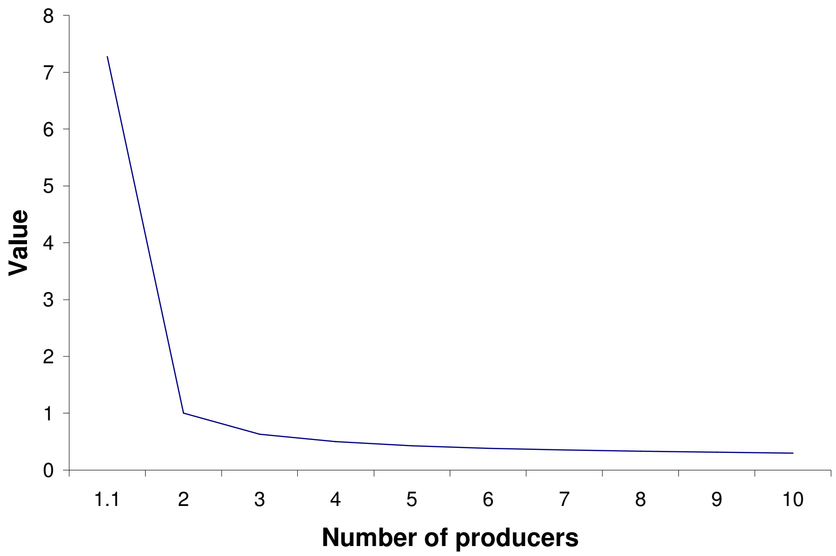

Figure 2.2 Value and the number of producers

2. Value and the number of producers

From (2.1), value is inversely related to the number of producers of a given product. Figure 2.2 displays the relationship between value and the number of producers. When the number of producers is small, the value of a product is high. That's why the products of monopolies and oligopolies are valued highly. If the base becomes one, i.e., absolute monopoly without substitution, value approaches infinity. This happens at some religious cults where only the spiritual leaders hold the key to heaven. In these types of organizations, the leaders often enjoy infinite power over their followers. The number of providers of most economic goods depends on many factors, some of which will be discussed in Chapter 3. In the following, we only give a brief discussion about the institutional structures that affect market entry and the number of suppliers for a given product.

Anti-trust regulations aim to prevent price fixing by existing providers of a service or product. They also intent to lower barriers to potential entry. Both measures, by increasing the number of choices, reduce the value of products, and hence the cost to consumers. For this reason, the value of a product will in general be lower in a more competitive market.

Patent rights and commercial secrets legislation, on the other hand, grant monopoly power and discourage the diffusion of knowledge. Patent rights and monopoly power allow the holders to maintain high product prices. Such a market virtually assures a firm's success. The social value embodied in such legislation encourages innovation, but discourages competition. The balance between fostering competition and protecting innovation so as not to stifle either is always a delicate one (Arrow, 1999).

It is often difficult to determine the exact number of providers of a service empirically. At the time of writing, Jetsgo, a Canadian airline, declared bankruptcy. There are three major operators in the air travel industry in Canada. They are Air Canada, WestJet and Jetsgo. There are some other regional carriers and international airlines competing for many routes. Together, we can assume four providers for the air travel service before Jetsgo declared bankruptcy. From (2.1), the value of each airline can be represented as

$$-\log_4 P \qquad \text{and} \qquad -\log_3 P$$

before and after Jetsgo declare bankruptcy. The change of value is therefore

$$(-\log_3 P)/(-\log_4 P) - 1 = \log_3 4 - 1 = 0.262$$

Jetsgo declared bankruptcy at the evening of March 10, 2005. The closing prices of stocks of WestJet and Air Canada at March 10 and 11 are 11.17, 15.6 and 32.19, 37 respectively. The price changes are

$$15.6/11.17-1=0.397 \qquad \text{for WestJet}$$

and

$$37/32.19-1=0.149 \qquad \text{for Air Canada}$$

respectively. The average change of price is

$$(0.397+0.149)/2=0.273$$

which is very close to the theoretical prediction of 0. 262.

Some theoretical and empirical results can be further refined. For example, this theory does not distinguish the sizes of different providers of a service. The refinement of the theory is left to the future research.

3. Substitutability and value

Many products and services are not identical but can substitute each other to a certain degree. The value of a single product can be defined as its entropy (2.2). The total value of two products, *X* and *Y*, can be defined as their joint entropy

$$V(X,Y)=-\sum_{j=1}^{n}\sum_{k=1}^{m}p_{jk}\log(p_{jk}) \qquad (2.3)$$

while the individual values of *X* and *Y* can be defined as

$$V(X)=-\sum_{j=1}^{n}p_j\log p_j$$

$$V(Y)=-\sum_{k=1}^{m}q_k\log q_k$$

It can also be proved that (Shannon, 1948)

$$V(X,Y)\le V(X)+V(Y) \qquad (2.4)$$

The equality holds only when X and Y are independent, i.e., X and Y are not substitutes or partial substitutes for each other. This means that substitutability reduces the value of a product, which is a very intuitive conclusion that is also verifiable via common sense observation in the marketplace. The purpose of brand name management and advertisement of a product is to make a product special and to reduce the perception and/or reality of substitutability for it, which increases the value of that product.

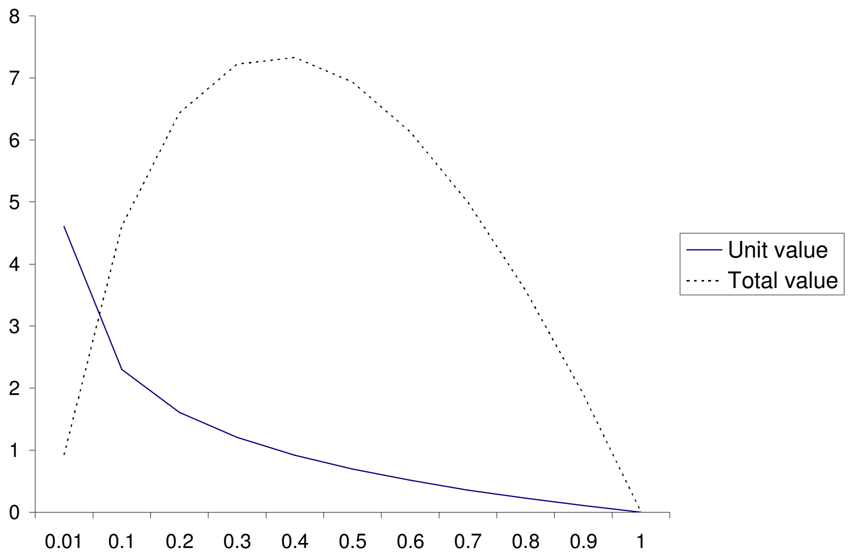

Figure 2.3 The unit value and total value of a product with respect to scarcity

4. Market size, product life cycle and product value

Suppose the potential market size of a product is M. The percentage of people who already have the product is P. Then the unit value of the product is

$$-\log P \quad (2.5)$$

Since the number of people who have bought the product is *MP*, The total value of the product is

$$MP(-\log P) \quad (2.6)$$

From (2.6), the value of a product is higher with a larger market size. Figure 2.3 is the graph of unit value and total value of a product with respect to its abundance. From Figure 2.3, we can explore the relation between the value of a product and product life cycle. When a product is new and scarce, the unit value is high. Its total value is low. As the production increases, the total value will increase as the unit value decreases. When the production quantity is over a certain level, however, the total value of a product will start to decrease as well. Intuitively, this is easy to understand. The market values of manufacturers of mature products are generally low, although the production processes are very efficient. This observation shows that efficiency is not equivalent to value.

The above discussion shows that the implications of identifying value with the reduction of entropy are highly consistent with our intuitive understanding of economic value. It should be noted that in economic processes, a final product embodies many different kind of scarcities: labor, raw materials and capital. A detailed analysis of the value of a particular product will be much more involved. For example, black and white television sets are less common than color television sets and yet they have less economic value. This is because the process of making color TV takes more scarce resources such as labor. The value of a final product is the sum of total scarcity.

2.3. Physical Entropy, Information and Economic Value

The discussion about the relation between information and physical entropy began with the paradox of Maxwell's demon (Maxwell, 1871). In 1870s, Boltzmann defined the mathematical function of entropy, which Shannon (1948) identified as information many years later. From the discussion in Chapter 1, information is the reduction of entropy, not only in a mathematical sense, as in Shannon's theory, but also in a physical sense. How then can economic value as entropy be linked to physical entropy?

From Chapter 1, both natural selection and sexual selection indicate that human beings favor low entropy sources. This observation offers a connection between the entropy theory of value and the subjective utility theory of value. "Mind is an organ of computation engineered by natural selection" (Pinker, 1997, p. 429). It calculates the entropy level and sends out signals of pleasure for accumulating and displaying low entropy and signals of pain for dissipation of low entropy. Jevons "attempted to treat economy as a calculus of pleasure and pain" (Jevons, 1957, p. vi). Pleasure is generally associated with the accumulation or display of low entropy level, such as the accumulation of wealth, and conspicuous consumption. Pain is associated with dissipation of low entropy, such as work and the loss of money. So value in subjective theory, as a measure for pleasure and pain, is intrinsically linked to the level of entropy.

The current economic theory states that the value of a commodity is a function of supply and demand. From the theory of natural and sexual selection, the demand of an economic commodity is driven by its level of entropy. The supply of an economic commodity is constrained by its scarcity, with entropy as its unique measure (Applebaum, 1996). Therefore the level of entropy offers a natural measure of economic value.

It is easy to understand the objective theory of value from the entropy theory of value. Since the entropy level of a system increases spontaneously, the reduction of entropy in a system represents the effort that has been made. Entropy level may be the closest to an invariant measure of value of labor and other commodities. While economic values of commodities are highly correlated with the level of physical entropy, they are not identical for several reasons. In the following, we will discuss two: One from the perspective of information theory and another from the institutional structures that regulate scarcity.

First, the entropy level we perceive of a commodity is different from its objective entropy level. From information theory, the amount of information one can receive, R, is equal to the amount of information sent minus the average rate of conditional entropy.

$$R = H(x) - H_y(x) \tag{2.7}$$

The conditional entropy $H_y(x)$ is called the equivocation, which measures the average ambiguity of the received signal (Shannon, 1948). From our discussion in Chapter 1, equivocation arises because receivers don't have the complete background knowledge of signals. For example, gold, a scarce commodity, is highly valuable. Another commodity could be as scarce as gold, but unlike shiny and stable gold, it could be very difficult to identify. Most people will not invest much effort to gain knowledge needed to identify this commodity because the cost outweighs the potential benefit. Thus, it registers less attention and is valued less by human beings.

Second, scarcity of a commodity is regulated by the institutional structures that enforce property rights. For example, the value of an invention is influenced by how long and how broad patent protection is granted. The value of a patent is higher in a system where patents are valid for twenty years than one for ten years. From (2.4), substitutability reduces the value of a product. If patent protection is defined broader, the value of an invention is higher. Economic value, as a function of scarcity, is to a great extent regulated by institutional structures. Among all the institutional measures that regulate scarcity, the most important regulation is the immigration laws that regulate the scarcity of labor forces, which makes persistent wage differential across regions possible.

2.4. The Entropy Theory of Value and Information

Because of the equivalence of entropy and information, an entropy theory of value is inevitably an information theory of value. Information is often regarded as a rather unusual commodity. In this section, we will show that informational and physical commodities share most of the fundamental properties from the perspective of entropy theory. Since Arrow (1999) offers a clear description about the special characteristics of information as an economic commodity, our discussion is based on his writing.

> The algebra of information is different from that of ordinary goods. … Repeating a given piece of information adds nothing. On

> the other hand, the same piece of information can be used over and over again, by the same or different producer(s). (Arrow, 1999, p.21)

From (2.7), the amount of information received is the information of source minus equivocation. Repeating a signal of information helps reduce equivocation. That is why the same commercials are repeated many times on TV. A more detailed analysis of commercials by a company, say Coca Cola, will illustrate the concept more clearly. Most commercials of Coca Cola spread the same information: Drink Coca Cola. The purpose of the commercials is to reduce the equivocation in information transmission between the sender, Coca Cola company and the receivers, the potential consumers. Usually same commercial will be repeated many times and different commercials are designed to relate the viewers to Coca Cola in different ways. However, the efforts of Coca Cola will not automatically reduce the equivocation between the sender and the receivers. Other soft drink companies and other matters in life compete for attention. As a result, the equivocation between Coca Cola and the general public may increase, despite the efforts from Coca Cola. From the thermodynamic theory that all low entropy sources have a tendency to diffuse, repeating the same piece of information is essential to keep it valuable. The essence of a living organism is to repeat and spread the information encoded in its genes.

It is often thought that the use of information does not involve rivalry, since "the same piece of information can be used over and over again, by the same or different producer(s)". This property is not confined to information. The same hammer "can be used over and over again, by the same or different producer(s)". However, the value of the same information will be different for different users or at different time. For example, if an unexpected surge of corporate profit is known by very few people, i.e., when P is very small and $-\log P$ is very high, this information would be highly valuable. Huge profit could be made by trading the underlying stocks. But when it is known to many people, the value of such information is very low. In general, when some knowledge is mastered by many people, its market value is very low.

> The peculiar algebra of information has another important implication for the functioning of the economic system. Information, once obtained, can be used by others, even the original owner has not lost it. Once created, information is not scarce in the economic sense.

> This fact makes it difficult to make information into property. It is usually much cheaper to reproduce information than to produce it in the first place. In the crudest form, we find piracy of technical information, as in the reproduction of books in violation of copyright. Two social innovations, patents and copyrights, are designed to create artificial scarcities where none exists naturally, although the duration of the property is limited. The scarcities are needed to create incentives for undertaking the production of information in the first place. (Arrow, 1999, p. 21)

Information is a type of low entropy source. Utilization of low entropy source from others is a universal phenomenon of living systems. "Once again animals discover the trick first. … butterflies, did not evolve their colors to impress the females. Some species evolved to be poisonous or distasteful, and warned their predators with gaudy colors. Other poisonous kinds copied the colors, taking advantage of the fear already sown. But then some nonpoisonous butterflies copied the colors, too, enjoying the protection while avoiding the expense of making themselves distasteful. When the mimics become too plentiful, the colors no longer conveyed information and no longer deterred the predators. The distasteful butterflies evolved new colors, which were then mimicked by the palatable ones, and so on." (Pinker, 1997, p. 501)

So the perceived uniqueness of copying information products in human societies is actually quite universal within living systems. Once we look at the living world from the entropy perspective, it can hardly be otherwise. In human societies, the attempt to copy and reproduce valuable assets, whether informational or physical assets, is also universal.

The fashion industry offers an example that illustrates the dynamics of innovation and copying clearly. When a new fashion style is created, it is scarce and hence valuable. This valuable information will then be copied by others. As more people copy the style, P increases, $-\log P$ decreases and the value of the fashion decreases. To satisfy the demands for high value fashions, new fashion styles "are designed to create artificial scarcities where none exists naturally".

Protection of an organism's source of low entropy to prevent access by others is also a universal phenomenon of living systems. Animals develop immune systems to protect their low entropy source from being accessed by microbes. Plants make themselves poisonous to prevent their low entropy from being accessed by animals. When space is a limiting factor in survival or reproduction, animals defend their territory vigorously (Colinvaux, 1978). Whether to enforce the property rights depends on the cost of enforcement and the value of the low entropy source. When information products become an important class of assets, the property rights of physical assets are naturally extended to informational assets.

2.5. Economic Wealth and Social Welfare

From the above discussion, it is clear that economic wealth and social welfare are two distinct concepts. Economic activities provide low entropy sources for the survival and comfort of human beings. From the second law of thermodynamics, the reduction of entropy locally is always accompanied by the increase of high entropy waste globally. So "externality" is not a form of "market failure" but a direct consequence of fundamental natural laws. Since low entropy product is more concentrated while high entropy waste is more diffuse, the economic value of a product is easier to measure than the harmful effects of the wastes. Usually, a product is developed to satisfy certain market demand. Its utility is easily appreciated by the customers, who are willing to pay for the product. This is the source of economic value. The harmful effects of the wastes, being more diffuse, affect more people but usually at very low level. These effects often take very long time to get noticed. When the human population density and consumption level is low, most of the high entropy wastes that humans generate are absorbed by microbes and other natural forces with little human effort (Margulis, 1998). This vital recycling business is accorded no economic value. As the population density and the level of consumption increases, however, direct human intervention is needed to move the high entropy waste away from where people reside. The economic value of the waste management business is a function of the level of effort invested by the public in the recycling of wastes. This value is not equivalent to environmental quality of human habitats. Since high entropy waste is more diffusive, the market of recycling businesses is generally created by legal and regulatory methods to prevent the degeneration of the environment.

Theories in institutional economics often suggest that externalities can be internalized with institutional measures. These measures, however, are achieved with costs. For example, companies often pursue internal stability in their working environment. However, more training and better pay packages are often required to maintain internal stability. In living systems, warm blooded animals are able to maintain stable core temperature near the biochemical optimum. However, they achieve stable internal environment with much higher energy costs of metabolism than cold blooded animals, whose internal environment fluctuates with external environment (Smil, 1999, p. 62).

While economic wealth is not equivalent to social welfare, economic value, as a reflection of human efforts, is generally geared toward human welfare over the short term. This is why economic prosperity is often consistent with the improvement of social welfare in a particular point of time. However, wealth, as low entropy of human society, is ultimately supported by low entropy from nature. In the last several hundred of years, world wide consumption of energy has been increasing steadily with the economic progress (Smil, 2003). Since our current civilization is based on fossil fuel, the eventual depletion of fossil fuel will shake the foundation of today's lifestyle.

In general, wealth represents the total dependence of each other in a society. The increase of one's wealth means the increase of the dependence of others on him and hence the increase of his power. While it is natural for an individual or a company to pursue strategies that maximize wealth, national and international policies often concern more about long term sustainability of ecological and social systems.

2.6. Concluding Remarks

Theories built on a sound physical foundation often provide simple and intuitive results on practical problems. Shortly after Shannon's work of 1948 that identified information as entropy, Weaver commented, "Thus when one meets the concept of entropy in communication theory, he has a right to be rather excited --- a right to suspect that one has hold of something that may turn out to be basic and important" (Shannon and Weaver, 1949, p. 13). The development of information theory in the last half century has proved his foresight. This entropy theory of value, which establishes an explicit link between economic value and physical entropy, offers an analytical theory that is highly consistent with our intuitive understanding of economic value.