MATH 101 Lab 4

In this lab we will draw curves defined by parametric equations and polar coordinates and find areas and arc lengths. The length of an arc of a curve given by parametric equations $x = f(t)$, $y = g(t)$, where $\alpha \leq t \leq \beta$ traces the curve once, is given by

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

A polar equation $r = f(\theta)$ can be written as a parametric equation in the xy-plane as $x = r\cos(\theta)$, $y = r\sin(\theta)$. The polar equation for arc length is

$$L = \int_{a}^{b} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta.$$

In order to plot the parametric curve $x = sin(t)$, $y = cos(t)$ with $0 \leq t \leq \pi$ we use the **plot** command

```
> with(plots);
> plot([sin(t),cos(t),t=0..Pi]);
```

To plot the polar equation $r = \cos(2\theta)$ with $0 \leq \theta \leq 2\pi$ we use the parametric equations $r = \cos(2t)$ and $\theta = t$ with $0 \leq t \leq 2\pi$. The `coords=polar` option tells Maple to use polar coordinates. The `scaling=constrained` option indicates that the two axes have the same scale.

```
> plot([cos(2*t),t,t=0..2*Pi], coords = polar, scaling=constrained);
```

Question 1

For $r = \cos(2\theta)$, what interval of θ gives the loops on the positive x-axis, on the positive y-axis, on the negative x-axis and on the negative y-axis? What is the area of one loop of the flower? Find the answer by using Maple and by doing it by hand. What is the arc length of one loop?

Question 2

Plot the polar equation $r = 1 + cos(\theta)$ with $0 \leq \theta \leq 2\pi$.

We can watch this graph being drawn with the animate command. Be sure to try out the buttons at the top of the plot to see what they do.

```
>  animate([1+cos(d*t),d*t, t=0..2*Pi],d=0..1,coords=polar);
```

Print the graph of $r = 1+cos(\theta)$ and label the points where $\theta = 0, \pi/4, \pi/2, \pi/3, \pi, 2\pi$.

Question 3

If we change the constant that cosine is multiplied by the graph looks different. Plot the polar graph of $r = 1 + 1.7\cos(\theta)$ on the the interval $0 \leq \theta \leq 2\pi$

We can animate the family of polar curves $r = 1 + c \cdot \cos(\theta)$ with the following commands.

```
> r:= t->1+c*cos(t) ;
> animate([r(t),t, t=-Pi..Pi],c=-3..3,coords=polar,frames = 20,
> scaling=constrained) ;
```

What value of c gives a circle? What value of c gives a cusp? What values of c produce a curve with two loops? What values of c produce a curve with only one loop?

Question 4

Use the polar curve given by $c = 2$ in the family of curves in Question 3. Animate the drawing of this curve for $0 \leq \theta \leq 2\pi$ as done in Question 2. Plot this curve and label several points on it. Find the area between the two loops. Find the length of the inside loop and the length of the outside loop. Label these values on the plot.

Hints

Commands that might be helpful for this lab included **diff**, **evalf**, and **solve**. Also, use functions rather than expressions.

Bonus

Derive the polar equation for arc length from the parametric equation for arc length.