FINAL EXAM

MATH 101, Winter 1999

Tuesday, April 13, 1999

NAME AND STUDENT NUMBER:

1. Write down all your work.
2. Calculators are allowed, but NOT NEEDED.
3. The exam is 3 hours.
4. This booklet contains 15 pages.
5. Maximum Possible Score = 70 (14 questions, 5 marks each)
6. Good luck and do your very best!

1. Sketch the region bounded by the curves $y = e^x$, $y = e^{-x}$ and $y = e^2$, and find the area of the region.

2. Find the volume of the solid obtained by rotating about the x-axis the region bounded by the curves $y = x$ and $y = x^2$.

3. A cylindrical hole of radius 6cm has been drilled straight through the center of a sphere of radius 10cm. Use the method of cylindrical shells to find the volume of the remaining solid.

4. Evaluate the integral

$$\int_0^1 x \tan^{-1} x \, dx.$$

5. Evaluate the integral

$$\int \cos^4 x \sin^3 x \, dx.$$

6. Evaluate the integral

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx.$$

7. Evaluate the integral

$$\int \ln^2 x \, dx.$$

8. Find the length of the curve defined as $x = 3t - t^3$, $y = 3t^2$, $0 \leq t \leq 3$.

9. What is the connection between polar and Cartesian coordinates? Show that $r = 6\cos\theta + 8\sin\theta$ is the equation of a circle and find its center and radius.

10. Find

$$\lim_{n\to\infty}(\sqrt{n^2+1}-n)\sqrt{3n^2+n}.$$

11. Find the values for which

$$\sum_{n=0}^{\infty} (2x)^n$$

converges. Find the sum of the series for those values.

12. Test for convergence

$$\sum_{n=1}^{\infty} \frac{2n+1}{3n^2-1}.$$

13. Test for convergence

$$\sum_{n=1}^{\infty} \frac{(n+1)2^n}{n3^{n+1}}.$$

14. Test for convergence

$$\sum_{n=3}^{\infty} \frac{1}{n^2 - 4}.$$