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AN APPROXIMATION FOR BEEKHUIS' MORTALITY MODEL

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SUMMARY

Beekhuis published in 1966, as part of his growth model for radiata pine stands, a graphical model for mortality. This mortality model has also been used, with slight modifications, as a component in later growth models for radiata pine and Douglas fir.

The construction of Beekhuis' mortality model is described and equations for it are given.

An approximation which is easier to use in computer implementations was obtained. It may be written as

$$S^c - S_0^c = (bH)^c - (bH_0)^c \quad ,$$

where S_0 and S are initial and final spacings, H_0 and H are initial and final heights, and b and c are constants.

Predictions of spacing with Beekhuis' equations and with the approximation differ by 4% in the worst possible case. The approximation has been used in several interactive computer implementations of the growth models.

AN APPROXIMATION FOR BEEKHUIS' MORTALITY MODEL

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1. Introduction

Beekhuis (1966) developed a graphical model for predicting natural mortality, as part of his growth model for Radiata pine. This model has also been used, with different parameter values, as a component in other growth models for Radiata pine and Douglas fir in Kaingaroa Forest, Southland and Auckland (Mensuration Project Team 1978).

The computer implementation of Beekhuis' graphical procedure is somewhat complex. A simple approximation has been developed and used in several growth model programs.

Section 2 describes Beekhuis' mortality model, and the approximation is presented in section 3. The accuracy of the approximation is evaluated in section 4.

2. Beekhuis' mortality model

Figure 3 in Beekhuis (1966), reproduced here as figure 1, describes the changes in average spacing (or stocking) with the height of the stand. He uses average triangular spacing, which is 1.074 times the average square spacing, i.e., $107.4/\sqrt{\text{stocking (sph)}}$. The relative spacing is the

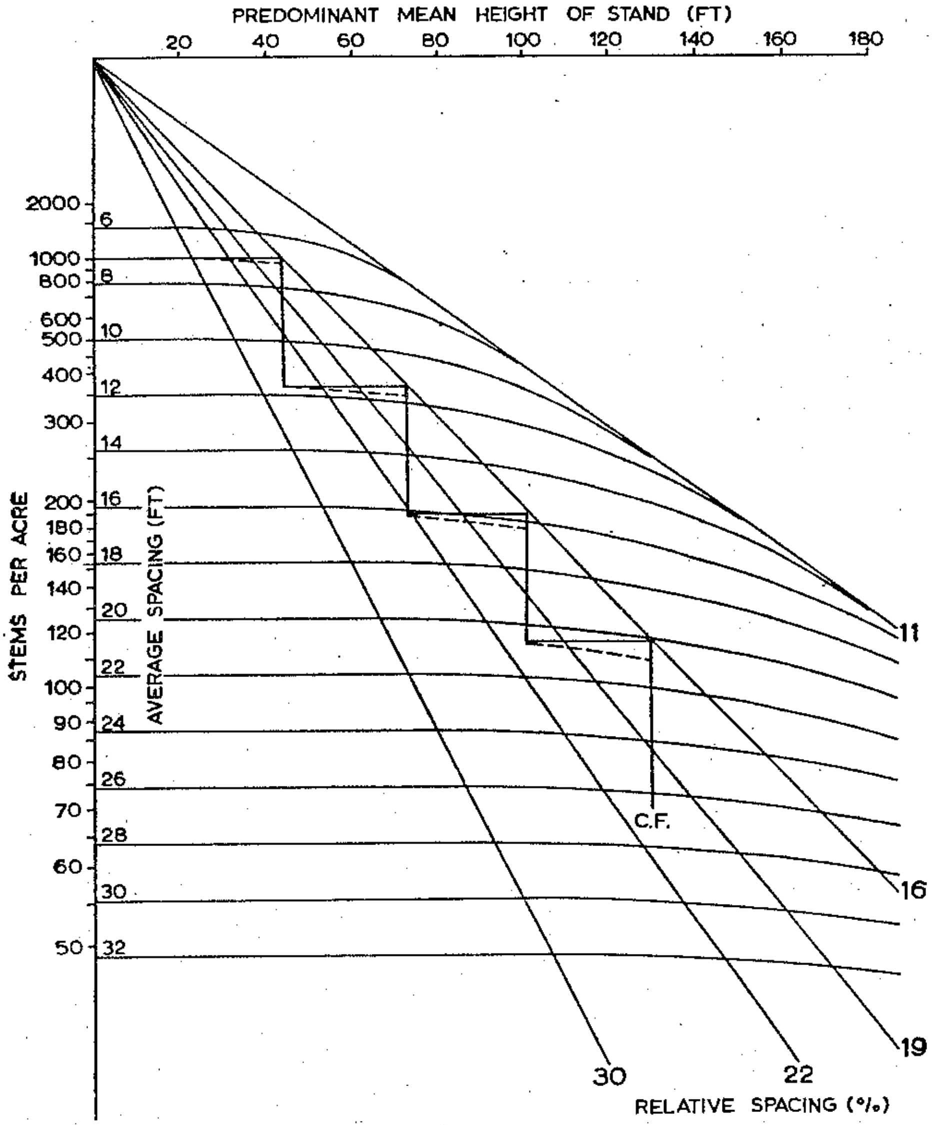


Fig. 3. Normal mortality trends. C.F. = clear felling.

FIGURE 1

ratio of average spacing to predominant mean height.

The graph was constructed as follows (Beekhuis, pers. comm.). A scale of ten units of height to one unit of spacing was used. For each spacing/height curve, a horizontal line was drawn from the spacing axis to the 30% relative spacing line. Between 30% and 11% relative spacing the curve follows a circle such that it is tangent to the 11% relative spacing line, and tangent to the horizontal at the point of intersection with the 30% relative spacing line (figure 2). To the right the curves coincide with the 11% line. The curves were then drawn at a different scale for producing his figure 3.

From the construction it is found that the equation for any spacing/height curve is

$$S = \begin{cases} S_0 , & H \leq S_0/a \\ S_0[1+r-\sqrt{r^2-(H/10S_0-1/10a)^2}\,] , & S_0/a \leq H \leq 10rS_0 \\ bH , & H \geq 10rS_0 \end{cases} \qquad (1)$$

where

$$r = \frac{1 - b/a}{\sqrt{1+(10b)^2} - 1} , \qquad (2)$$

and S = average spacing

H = predominant mean height

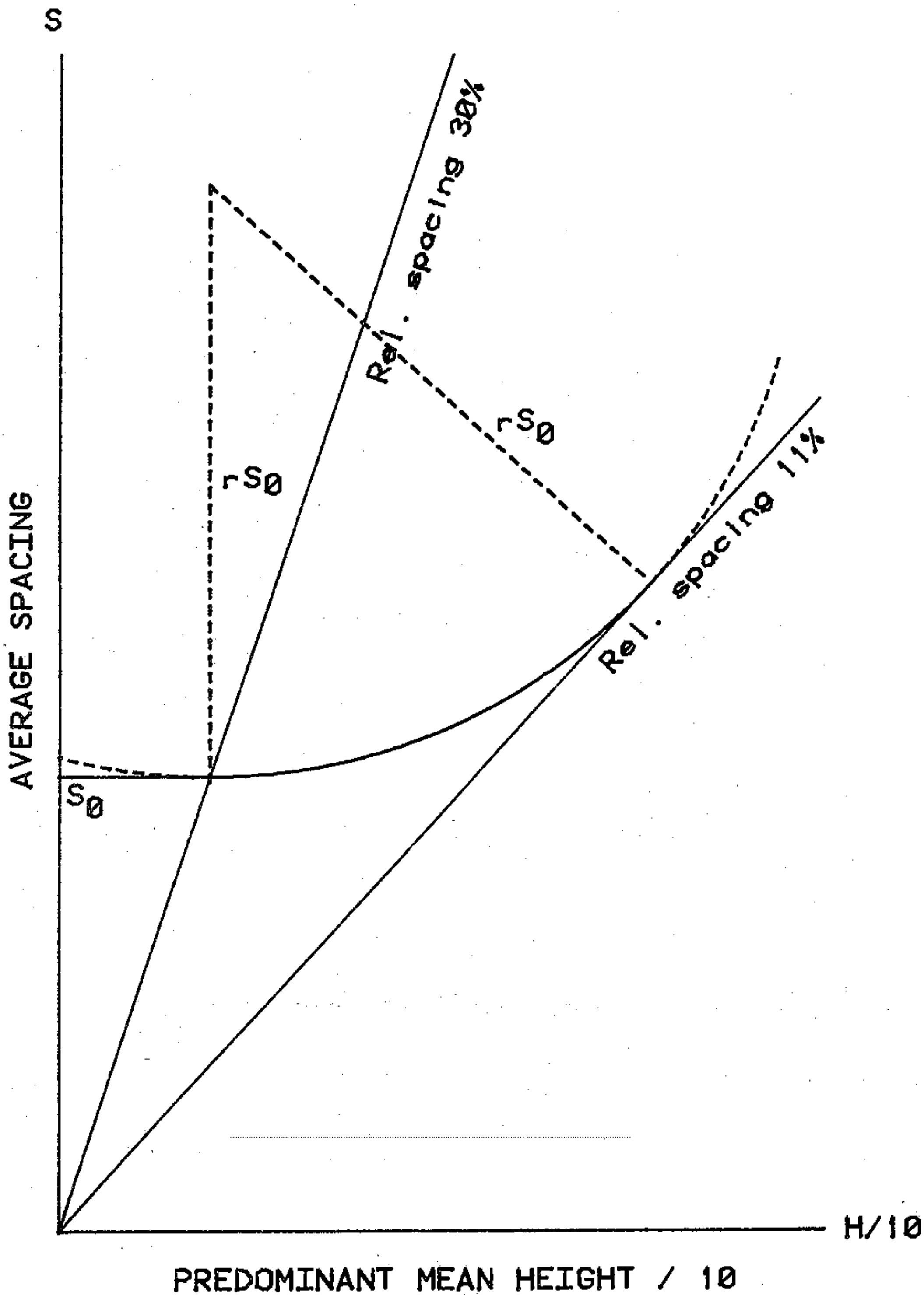

S
AVERAGE SPACING
rS0
rS0
Rel. spacing 30%
Rel. spacing 11%
S0
H/10
PREDOMINANT MEAN HEIGHT / 10

FIGURE 2

S_0= initial spacing

a = 0.3 = relative spacing (decimal) at which mortality begins

b = 0.11 = minimum relative spacing.

This mortality model has also been used in growth models for Radiata pine and Douglas fir in Kaingaroa Forest and for Radiata pine in Southland and Auckland Conservancies, with different values of a and b (e.g. a=0.2 and b=0.1 for Kaingaroa Radiata) and using top height instead of mean predominant height (Mensuration Project Team 1978). The Fortran programs prepared by the Mensuration Project Team contain a subroutine based on equations similar to (1) and (2).

The fact that the spacing/height culve takes different forms for different values of height and spacing complicates the computation of changes in stocking for given changes in height.

3. An approximation

It is easy to see that in Beekhuis' model the slope of the curves of S over H depends only on the relative spacing R=S/H. The curves then satisfy a differential equation of the form

$$\frac{dS}{dH} = f(R) , \qquad (3)$$

with f(R) increasing from 0 to b as R decreases to the limiting relative spacing b. An equation of the form

$$\frac{dS}{dH} = R(b/R)^c \quad , \tag{4}$$

was found to give a reasonable fit, and at the same time is easily integrable.

Integrating (4) we get the approximation

$$S^c - S_0^c = (bH)^c - (bH_0)^c \quad , \tag{5}$$

where H_0 and H are the initial and final heights and S_0 and S are the initial and final spacings. The exponent c depends on a and b, and has been selected so that, starting from the same spacing S_0 at H_0=0, (1) and (5) coincide for $H=S_0/b$. For this value of H (5) gives

$$S = S_0 \cdot 2^{1/c}$$

and (1) gives

$$S = S_0[1+r-\sqrt{r^2-(1/b - 1/a)^2/100}\,] \quad ,$$

from where

$$c = \ln 2 \;/\; \ln\,[1+r-\sqrt{r^2-(1/b - 1/a)^2/100}\,] \quad , \tag{6}$$

where r is given by (2). This is just a convenient way of obtaining c for various values of a and b; other values might give better approximations.

For the models described in Mensuration Project Team (1978):

Model	a	b	c
BEEK (Beekhuis 1966)	0.30	0.11	5.501
KGM1,KGM2,AGM1,SGM1	0.20	0.10	6.734
DFIR	0.18	0.09	6.594

Some alternative forms of (5) are:

$$S = [S_0^c - (bH_0) + (bH)^c]^{1/c} \qquad (7a)$$

$$S = S_0[1- (bH_0/S_0)^c + (bH/S_0)^c]^{1/c} \qquad (7b)$$

$$N^{-c/2} - N_0^{-c/2} = (bH/107.4)^c - (bH_0/107.4)^c \qquad (7c)$$

$$N = N_0[1-(bH_0\sqrt{N_0}/107.4)^c + (bH\sqrt{N_0}/107.4)^c]^{-2/c} , \qquad (7d)$$

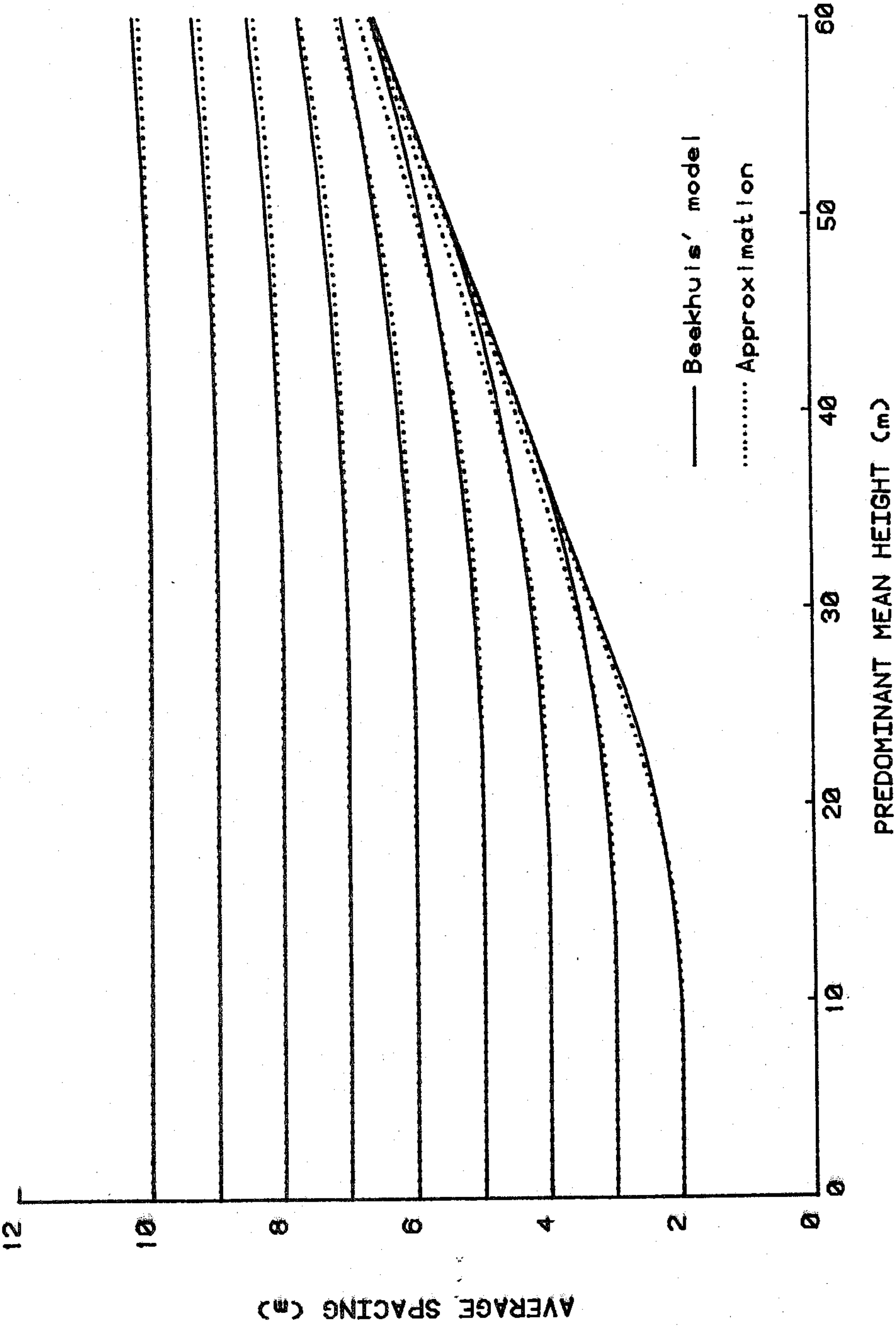
etc., where N is the stocking (sph) and H is in metres. Equations (5), (7a) and (7a) hold for any system of units, and may be used with square spacing if b/1.074 is substituted for b.

4. Accuracy

Figure 3 compares Beekhuis' curves computed from (1) with their approximations from (5), starting from the same initial spacings. The percent error is shown in figure 4.

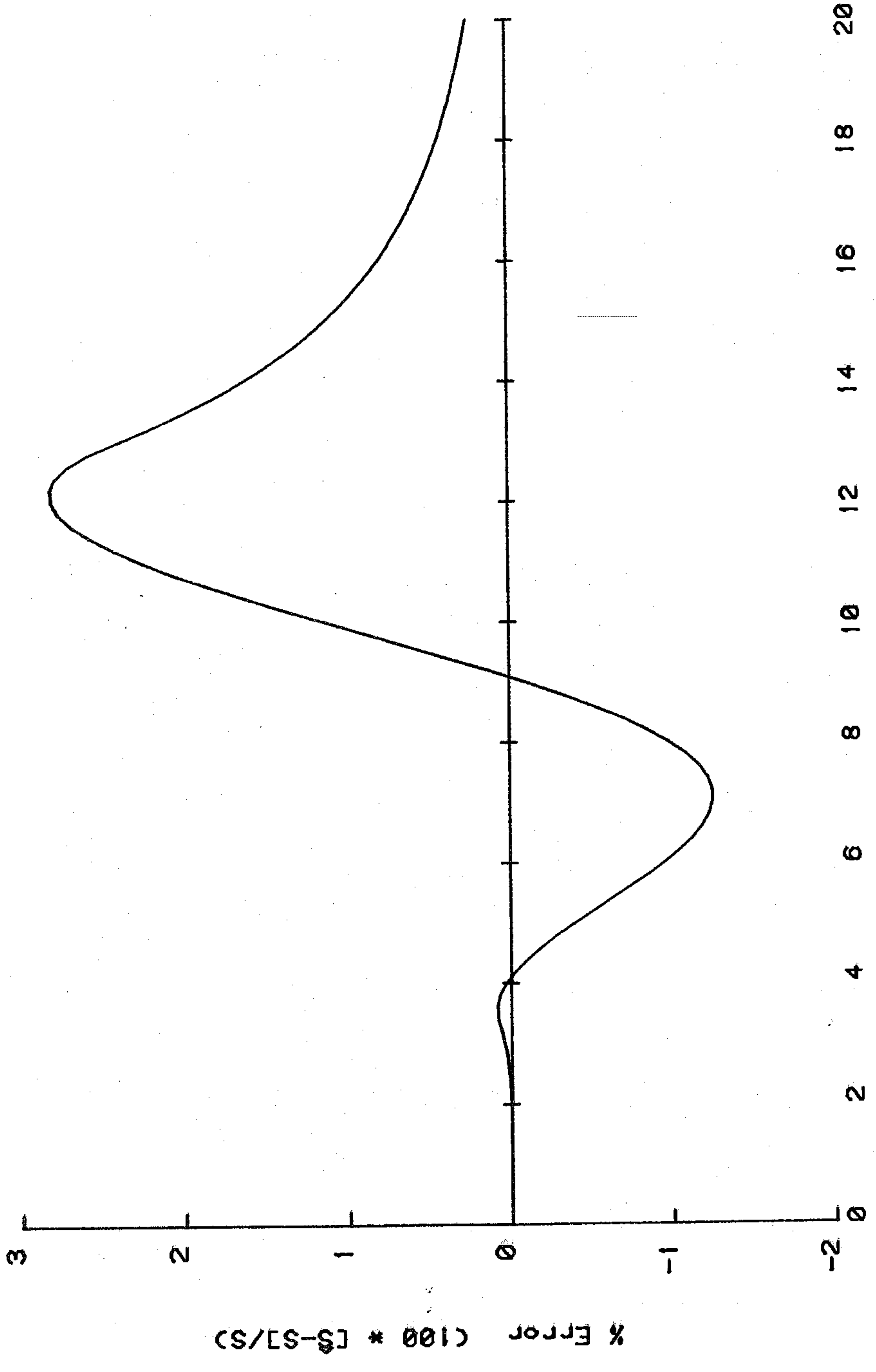
The error for the change in spacing for a given height interval depends on the location of the starting and end

FIGURE 3



Beekhuis' model
Approximation
PREDOMINANT MEAN HEIGHT (m)
AVERAGE SPACING (m)
0
10
20
30
40
50
60
0
2
4
6
8
10
12

FIGURE 4



points of the interval. From figure 4 it is found that in the worst case the error is about 4%, although in most cases the difference would be considerably less. The percent error for stocking is approximately twice as much. Similar results hold for the other growth models.

Considering the errors involved in estimating mortality, equation (5) seems satisfactory as a substitute for Beekhuis' mortality model. This approximation has been used in several interactive implementations of the growth models.

REFERENCES

Beekhuis, J. (1966) Prediction of yield and increment in Pinus Radiata stands in New Zealand. Technical Paper No.49, Forest Research Institute, New Zealand Forest Service.

Mensuration Project Team (1978) Stand Growth Models. New Zealand Forest Service.