

Growth of Thinned and Pruned Stands *

Oscar García, Forest Research Institute, Rotorua, New Zealand

Abstract

General patterns in the growth of thinned and pruned stands are explored with the help of empirical growth models and graphical analysis of raw data. A variable representing relative stand closure makes possible relatively simple descriptions of stand development and silvicultural responses. It is commonly thought that gross volume increment in closed stands decreases with age, and is independent of stocking. It was found, however, that the increment does not vary significantly within the range of ages studied, but decreases with increasing tree spacing. The increment in an open stand relative to that of a closed stand with the same spacing is non-linearly related to the “degree of closure”. Descriptions of the change in the degree of closure caused by thinning and pruning, and of the subsequent rate of recovery, complete the information required to predict the effects of silvicultural treatments. These concepts may contribute to improve the accuracy of growth models, to more robust economic evaluations of silvicultural regimes, and to growth modelling in data-poor situations.

KEYWORDS: growth; growth models; thinning; pruning; radiata pine.

Introduction

There is a long history of research on the thinning and pruning of even-aged stands, especially in Central and Northern Europe. Good reviews in English can be found in Baskerville (1962), and Möller (1954). See also Braathe (1957), Möller (1960), and Baker (1950, Chapter 20). It is generally concluded that

within a “wide range” of stand densities, thinning has little or no effect in total gross production of stem volume. Thinning practice in New Zealand, however, often works well outside of this “wide range”. In addition, pruning intensity is often high enough to significantly reduce growth. This is easy to justify economically, since there must be a trade-off between volume production and increased product unit value. Rational decision-making then, calls for the ability to predict growth for a wide variety of silvicultural regimes, including very open stands.

Most growth models used for management predict growth as a function of stem characteristics, such as basal area, height, trees per hectare, diameter distributions, etc. (García 1988b). This is usually satisfactory, except following extremely heavy thinning and/or pruning, where the stand may not recover full use of the site potential for several years. The assumption that site occupancy is a function of tree size and stocking (Baskerville 1962) has limitations. Two stands with the same stem characteristics may differ in site occupancy if one has just been thinned and/or pruned while the other has not been recently treated. West *et al.* (1982) recognised this, and used the sum of the crown lengths per hectare as a measure of stand closure or site occupancy. Other measures have been used in more recent growth models (García 1988a, 1989).

An extensive data base of permanent sample plot data for radiata pine, covering a wide range of conditions, is available in New Zealand. This made possible to approach the development of growth models in the spirit of data analysis. That is, the emphasis has been in describing the data with flexible empirical models, not constrained by preconceived ideas. With the accumulated experience, and good descriptions of the actual behavior of forest stands, it may be appropriate now to look for general patterns and

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simpler characterisations of forest growth.

In this paper, a brief description of our stand growth models, and some ideas of closure and occupancy that have been found useful, are presented first. These concepts are then illustrated with raw data and relationships from the Pumice Plateau model. The potential for the development of simpler and more robust models is indicated.

Background

For a stand in a particular site and at a given point in time, growth and mortality can be regarded as functions of the current state of the stand. In a model, the state description must be complete enough to predict stand development with sufficient accuracy. At the same time, in models to be used for forest management, adequate estimates for the necessary variables must be obtainable at reasonable cost (García 1988b). In moderately homogeneous even-aged stands, models based on a few stand-level state variables have been found satisfactory for most silvicultural purposes.

Over the past 10 years, a series of regional growth models for radiata pine has been developed in New Zealand (Goulding 1986, García 1988a). These models describe the state of a stand by its basal area, number of trees per hectare, and top height, with some of them adding a fourth state variable representing “stand closure” (in one instance the foliar phosphorous content is also included to model the effect of fertilising).

Basal area, trees per hectare and height may be expected to describe well the state of stands subjected to moderate thinning and pruning regimes. Following a heavy thinning, however, the residual trees are not able to make immediate use of all the additional resources made available to them by the removal of competitors. Therefore, the growth of a stand after a heavy thinning is generally less than that of a stand of similar basal area, number of trees and height, but not recently thinned. The “closure” variable, discussed in detail below, attempted to account for this temporary less than full site occupancy. The effect was always relatively small, and not apparent in some of the data sets.

A similar loss of site occupancy should result from heavy pruning, until the stand recovers its full canopy. Ideally, this effect should be assessed, and if necessary modelled, using green crown level or crown length data (García 1979). Except for the latest Pumice Plateau model, this was not possible because of the lack of reliable information on these variables, so that any effects of pruning are confounded with the thinnings and other variables. West *et al.* (1982) used crown lengths from silvicultural trials in the Central North Island in a model for young radiata pine.

The recently completed Pumice Plateau growth model (PPM88) made use of the largest data set in the series. It is based on some 2000 increment periods in almost 300 permanent sample plots from the Kaingaroa and Tarawera forests in the Central North Island. The data covers a wide range of treatments (García 1988a), and it was possible to obtain adequate information on the thinning and pruning history of most plots. This model differs from the previous ones in three main aspects:

- a) Both thinning and pruning are modelled through a closure variable;
- b) Closure values are also used to model the growth of young stands, before canopy closure;
- c) Closure enters the growth equations in the form of multipliers, allowing for more consistent extrapolations (García 1989).

The performance of this model, and that of an earlier version without a closure variable (KGM3), are very satisfactory. The differences in growth forecasts between PPM88 and KGM3 are relatively small, except in the case of combinations of heavy thinning and pruning. Although both models can be suitable for yield forecasting in long term planning, the additional accuracy of PPM88 is important when comparing alternative silvicultural regimes.

The data and some relationships from PPM88 will be used here to investigate some interesting growth patterns. This data is from fairly good sites, where radiation is likely to be the main limiting factor. Different patterns may occur where growth is limited by the availability of water or nutrients.

Stand closure and site occupancy

What follows is a highly simplified interpretation of the mechanisms of stand closure and site occupancy. It is hoped, however, that these approximations are good enough for them to be useful, considering the current state of knowledge and the costs of detailed measurement.

Let us call a stand “closed” if it makes maximum use of the site resources, and “open” otherwise. I will use two variables to describe and explain the differences in growth between open and closed stands: The relative degree of stand closure, or “closure” for short, represents the amount of assimilating materials relative to that in a closed stand. The relative site occupancy, or “occupancy”, is the current gross volume increment of a stand relative to the increment that it would have if it were fully closed.

The closure may be thought of as the amount of foliage (in mass, leaf area, or tons of chlorophyll per hectare, for example), as a percentage of the equilibrium amount corresponding to the carrying capacity of the site. It might also include root biomass, etc. A more precise definition is not needed here. In a growth model the closure can be treated formally as an unobserved variable. It is only necessary to have an initial value, and relationships describing how it changes with time, thinning and pruning, and how it relates to occupancy (García 1989).

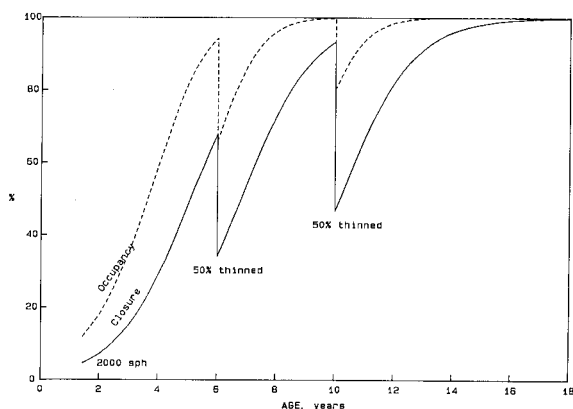


Figure 1: Example of changes in closure (solid line) and occupancy (dashes) with age and thinning.

It is assumed that closure starts at a small value proportional to the number of trees per hectare at the time of planting, and gradually increases up to a value of 100% when the stand closes. A thinning reduces the current closure in proportion to the percentage of basal area or of volume removed. Pruning also reduces closure. After thinning or pruning the stand closes again with the closure tending to 100% (see the solid line in Figure 1).

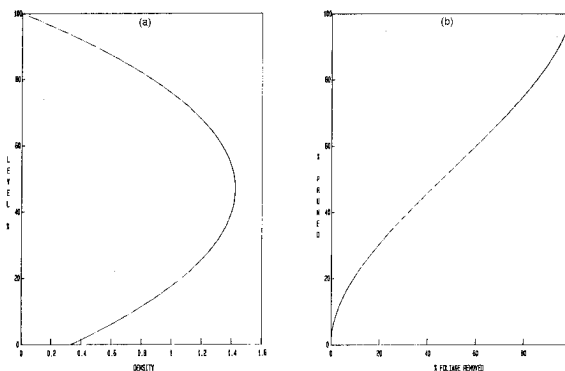


Figure 2: (a) A vertical distribution density of foliage in the canopy. (b) The cumulative distribution, relating percentage of foliage to percentage of canopy depth removed by pruning.

The effect of pruning on closure is more complex than that of thinning, where it is natural to assume that the percentage of assimilating material removed is close to the percentage of basal area or volume removed. Pruning is usually specified by pruning height. We can reason that a given vertical distribution of foliage within the canopy (Figure 2a) implies a corresponding relationship between percentage of canopy depth pruned and percentage of foliage removed (Figure 2b). Admittedly, this is somewhat crude. The distribution of foliage at the time of first pruning is likely to be different from that at a second or third lift. Also, the effect of removing foliage from below the canopy by pruning may be different from that of removing it from “within” the canopy by thinning, apart from the different impacts of thinning and pruning on root competition. Figure 2b was estimated indirectly in PPM88 from the effect on growth of prunings relative to thinnings (canopy depth was

defined as top height minus green crown level).

It is obvious that, other things being equal, total stand growth decreases with decreasing closure. It is also well known that moderate opening of a stand has a very small effect on growth, so that the relationship between occupancy (relative growth) and closure is not linear. To be specific, we can think of the change in photosynthesis as we add foliage at the base of a uniform canopy. Total photosynthesis increases as we increase the canopy depth. The increase becomes progressively less, however, as the light available to the additional leaves decreases. At some point, the light level becomes too low to compensate for the maintenance respiration of the marginal leaf and supporting branches, and the stand is considered to be fully closed. Presumably, closed stands would maintain a dynamic equilibrium through the formation of new leaves, and the death of those that cease to make a positive net contribution.

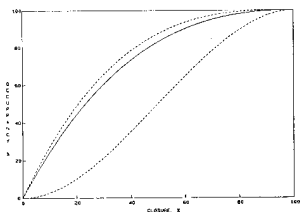


Figure 3: Three hypothetical relationships between occupancy and closure.

Similar arguments could be advanced for different canopy architectures, and for the relationship between root biomass and capture of soil resources. Whatever the details, reasonable relationships between closure and occupancy are illustrated in Figure 3. The horizontal tangents at full closure reflect the negligible net contribution of some of the leaves and/or roots. The dashed curves are the boundaries of a range of curves considered for PPM88, and the solid curve was the resulting estimate (García 1989). (Actually, this curve does not correspond exactly to occupancy as defined here, being based not on volume increment, but on the increment of a different function of basal area, stocking and height.) Using this, the occupancy for the example of Figure 1 would vary as indicated by the dashed curves in that figure.

West *et al.* (1982) had used successfully a measure of stand closure as the main driving variable in their EARLY growth model. It was defined as the product of the mean crown length and the number of trees per hectare, expressed in kilometres per hectare. The following main differences between this crown/ha value and the closure defined here may be mentioned:

- a) The reduction in crown/ha by thinning is proportional to the percentage of trees removed, instead of to the percentage of basal area or volume;
- b) Crown/ha decreases linearly with the length of crown pruned, instead of non-linearly as described in Figure 2b;
- c) The rate of recovery of crown/ha after thinning/pruning is equal to the height growth rate.

In PPM88 the closure is a variable that is not observable at all times. Nevertheless, given an estimated initial closure for seedlings, the curve in Figure 2b, and a relationship for the rate of increase in closure, the closure at any time can be derived from the stand pruning and thinning history (Figure 1). Alternatively, it is possible to start from a point in time when a stand might be regarded as approximately 100% closed.

As already said, these concepts of closure and occupancy have been useful in modelling the effect of heavy thinning and pruning, and the growth of young stands in PPM88. They were also used for thinning in some of the other models. The relationships involved were estimated indirectly, from their combined effect on stand development. Here, I will use these ideas to analyse the growth of thinned and pruned stands, largely by examining the raw data. The growth of fully closed stands is investigated first. Then the relationship between occupancy and closure is explored through the growth reduction immediately following thinning and pruning. Finally, the rate of closure recovery and the potential use of these ideas for silvicultural regime evaluation are discussed.

1 Growth of closed stands

The basic data is that used in the development of the Pumice Plateau model, PPM88 (see García 1988a for graphs of the basal area, stocking, and height coverage). Plots with site indices in the top and bottom 10% were excluded, leaving a range of site indices from 28 to 37 metres. The ages have been “normalised” to the equivalent of site 32 by scaling with a site-dependent factor which in the model had eliminated site differences. Unadjusted data shows similar patterns, but with a wider scatter of points.

I will examine growth in total stem volume per hectare. Volume gave more interesting results, and it might be expected to be more strongly related to assimilation than basal area, for example. It is not always realised that in general, the behavior of volume and basal area growth are markedly different, as demonstrated by Wiederman in 1931 (Braathe 1957, p.42).

The volume per hectare used here was estimated from basal area and top height by a simple stand volume equation. In the initial version of this paper I used the volumes given by the Permanent Sample Plot system, which are calculated from individual-tree volume tables and regression on a sub-sample of the plot trees. The results are similar, but the PSP volumes have a higher variation.

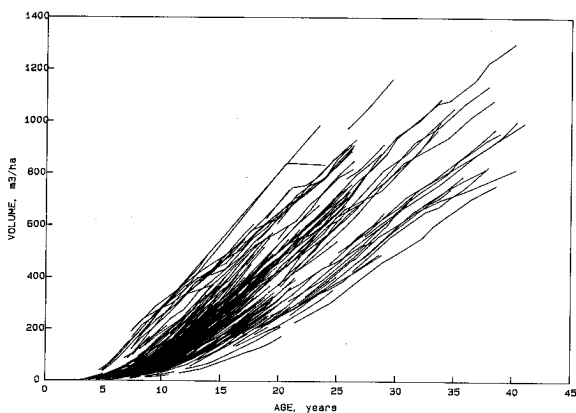


Figure 4: (a) Changes in total volume per hectare with age.

Figure 4a shows the volume measurements over

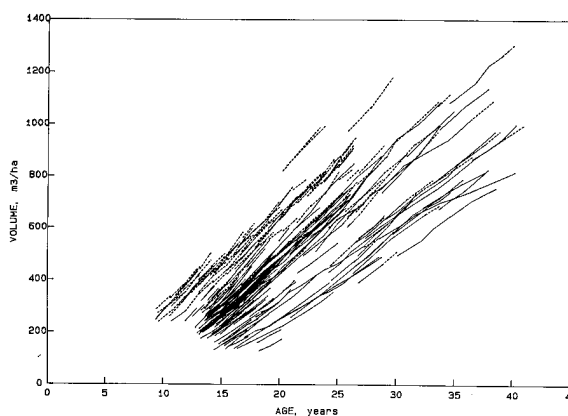


Figure 4: (b) Same, for gross volume in closed stands; dashed lines include natural mortality.

age. Pairs of successive measurements with no thinning or pruning between them are joined by straight lines. In Figure 4b, only measurements for “closed” stands are included (closure greater than 98%, as estimated by PPM88). In addition, in case of tree mortality the volume of the dead trees was added to the second measurement, and the pair joined by a dotted line to indicate that the slope represents a gross volume increment.

The trends of volume over age for closed stands are approximately linear within this range of ages and, as expected, growth tends to be slower in the less than fully closed stands. A slight divergence in the trends in Figure 4b may be noticed.

Annual volume increments for the closed stands were computed, using consecutive measurements with intervals from one to three years. As suggested by the linearity in Figure 4b, the increments show no obvious relationship with age (Figure 5a). The strongest relationship with any single variable is with the number of trees per hectare. Although the points are widely scattered, a roughly linear decrease in growth with increasing average spacing is observed (Figure 5b). The residuals from the regression line of Figure 5b are plotted against age in figure 5c, confirming that the (gross) volume increment is approximately constant, at least below commercial rotation ages.

The constancy of the volume increment may be

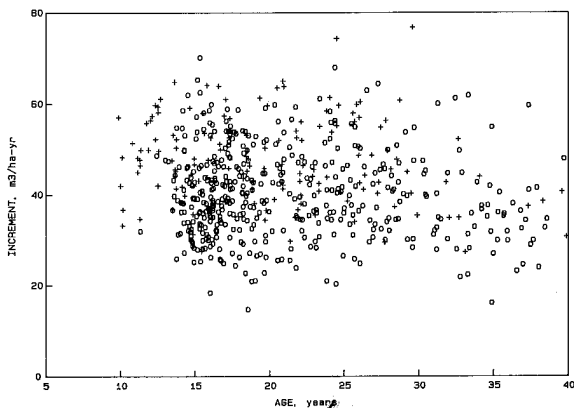


Figure 5: Observed volume increments, cubic metres per hectare-year. Crosses include mortality. (a) Increment vs age.

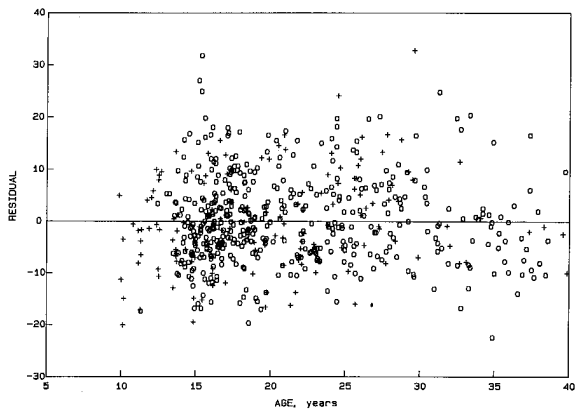


Figure 5: (c) Residuals from the line in (b) vs age.

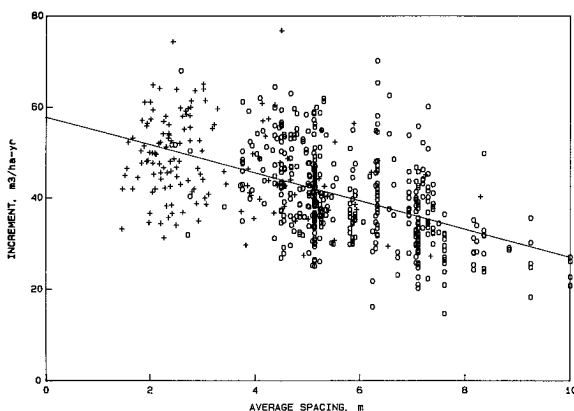


Figure 5: (b) Increment vs average square spacing.

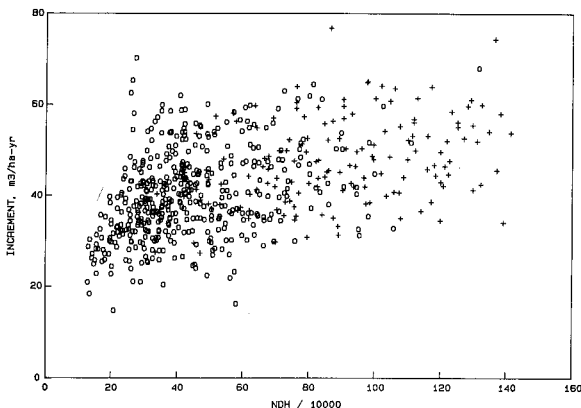


Figure 5: (d) Increment vs the product of stocking, mean dbh, and top height, an approximation to the bole surface area per hectare.

somewhat surprising. It is generally stated that, after culminating with canopy closure, the gross volume increment decreases with age. This is usually based on theoretical considerations, a typical argument being that respiration losses are roughly proportional to the surface area of stem and branches, which increases with age (Möller 1947; Baker 1950, p.370; Baskerville 1962, p. 55; Jarvis and Leverenz 1983). If the bole surface area per hectare is a good proxy for the total surface area, growth should decrease with the product of mean diameter, height and number of trees (Baskerville 1962, García 1974). Figure 5d indicates that this is not the case. Relying on

mechanistic arguments without supporting data can be risky.

Baskerville suggests that an increase of branch surface area with spacing might partially compensate for the decrease in stem surface area. It is also plausible that the efficiency in using extra growing space decreases with the distance from the stem. At any rate, the data suggests that the assumption that the gross volume increment of closed stands varies only with spacing is a reasonable approximation. Figure 6, from a review of spacing experiments by Sjolte-Jørgensen (1967), seems to confirm this. Obviously, the net increment will decrease as mortality sets in.

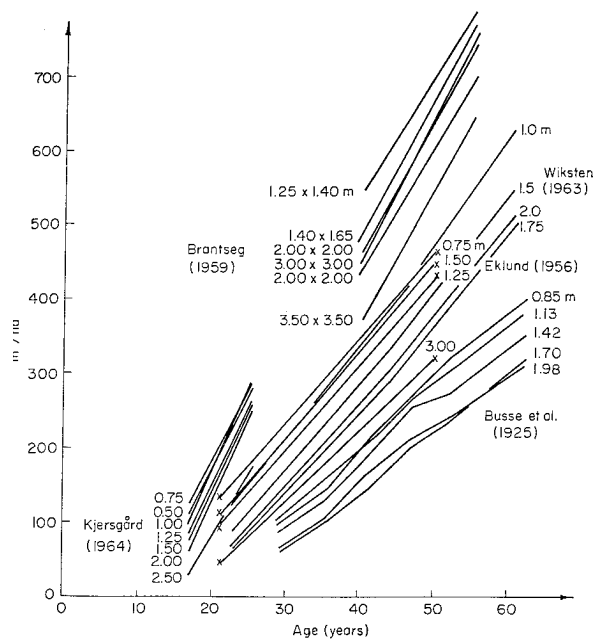


Figure 6: Total volume production as a function of age for various spacing experiments. Taken from Svolte-Jrgensen (1967).

Opening the stand

To examine the effect of closure on growth, cases with one measurement immediately following a thinning or pruning and another one year later were selected, and the annual volume increment was calculated. Closure was reduced by thinning in proportion to the percentage of basal area removed, and by pruning according to Figure 2b. The closure before treatment had to be estimated with PPM88. Therefore, the values obtained depend somewhat on the assumed rates of closure illustrated in Figure 1.

The increments and estimated closures after treatment are plotted in Figure 7a. The points shown as circles had more than 90% closure before treatment, so that they are less affected by the assumptions on rate of closure.

In Figure 7b, the degree of site occupancy is the increment as a percentage of the fully-closed increment estimated from the line in Figure 5b. The solid curve from Figure 3 is included as a reference. This

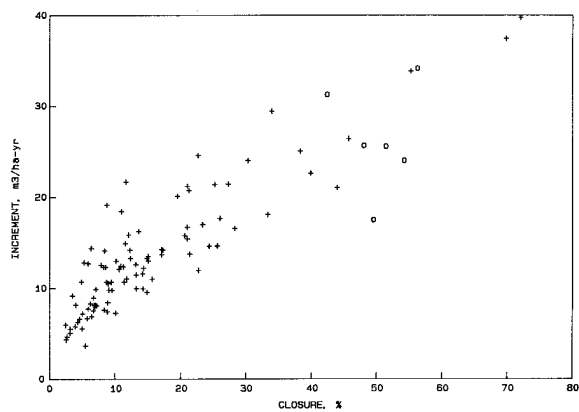


Figure 7: Volume growth in the first year following thinning and/or pruning. (a) Increment, cubic metres per hectare-year.

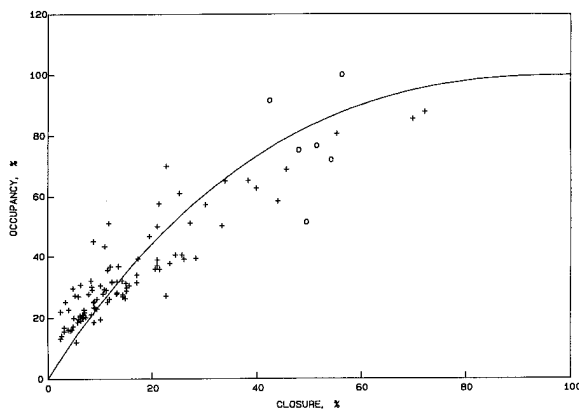


Figure 7: (b) Relative to the growth in a closed stand according to the line in Figure 5b. The solid curve from Figure 3 is shown for comparison. For the points, the closure is that at the beginning of the year.

graph gives some idea of the loss of growth that might be expected in the first year following treatments of various intensities.

How fast does a stand close?

In the previous section the loss of growth in the first year after thinning and/or pruning was demonstrated. To evaluate fully the effects of treatment

it is necessary to know also how fast the stand recovers closure and occupancy. I have not found any clear way of displaying this process using raw data. Sequences of increments are too variable to reveal distinct trends. An analysis of the curvature of the volume-age trends in young unthinned stands might help.

Clearly, the rate of closure, degree of closure, and occupancy in sample plot data are interrelated. Their values can only be estimated efficiently by estimating parameters simultaneously in an appropriate model. The rates of closure obtained in PPM88, shown in Figure 1, appear intuitively reasonable (the rate is a function only of the current closure). An alternative would be to use detailed series of measurements of crown cover, amount of foliage, or light interception. A great deal of work has been done on the relation between amount of foliage and assimilation (e.g. Jarvis and Leverenz 1983, and older work discussed in Baskerville 1962). However, not much information seems to be available on the dynamics of the canopy after alteration by management. This would still ignore below-ground processes.

The effect on final yield

In comparing two different treatments, the difference in final gross yield can be separated into two components. One results from differences in site occupancy, and could be derived from the areas above the occupancy curve (see Figure 1). The other component arises from the accumulation of any difference in the long-term growth rates.

As a simple example, let us compare the second thinning in Figure 1 with the alternative of no thinning. Assume that the stockings before and after thinning are 600 and 280 stems/ha, respectively, corresponding to average spacings of 4.1 and 6.0 m. The area above the occupancy curve in Figure 1 due to the thinning is roughly 15% of a 1-year increment, that is, a loss of about 6 m³ (from Figure 5b). Figure 5b indicates a difference of 5.8 m³/ha-yr in the long-term increment, which would result in a 87 m³ difference at age 25, assuming no mortality. To this we need to add the volume removed in the thinning. For the thinning to be profitable, the volume loss

must be compensated by the price differential for the larger mean tree size, after including the capitalised cost/revenue from the thinning.

Notice that the relationships used here are crude, and the numbers should not be taken too seriously. It is apparent, though, that eventually this kind of approach could complement the use of more complicated economic evaluation models. It allows focusing on the fundamental aspects, without being confused by less relevant assumptions.

Discussion

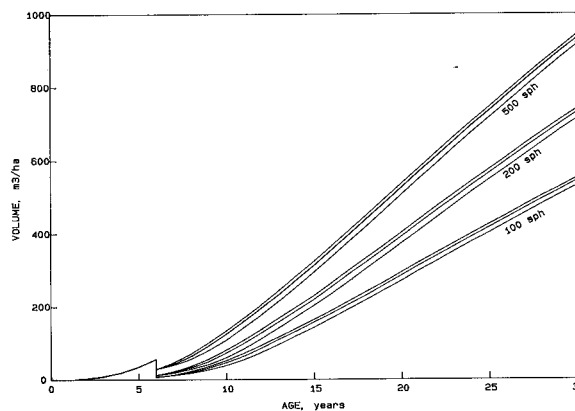


Figure 8: Some volume projections calculated with the Pumice Plateau growth model (PPM88). Site 32, planted at 1000 stems per hectare, one thinning and pruning at age 6. Thinnings to 100, 200, and 500 sph. Unpruned, and pruned to 4 and 6 m.

Figure 8 shows some growth projections computed with the PPM88 model. Although based on an unconstrained fitting of flexible empirical equations, the model behavior agrees with the simple results on the growth of closed stands obtained here. After closure, the volume curves are almost straight lines, with slopes depending on the spacing (some curvature at the top may be attributed to mortality). Lines at different levels, for the same spacing, are nearly parallel.

I have focused on volume increment, without discussing height growth or mortality. There is much evidence that top height is little affected by stand den-

sity “within wide limits” (Baskerville 1962, Beekhuis 1966). However, further investigation in radiata pine under extreme treatments, specially pruning, would be desirable. Mortality is usually not important in heavily thinned stands, although it needs to be included in a general-purpose model. Natural mortality is highly variable. Simple theoretical models such as the “3/2 self-thinning rule” (Weller 1987) do not fit well our data.

It is obvious from the graphs that variability in the data is high. Part of the variation is due to measurement and sampling error, but a large part of it arises from year-to-year weather differences. Scandinavian studies indicate standard deviations of about 12% in the annual diameter growth due to weather variation (Thammincha 1981). This is more troublesome in fast-growing species, where changes occur quickly and there is no time for the fluctuations to be smoothed-out over several years. Not been able to forecast the weather years ahead, it seems unlikely that additional variables or more complex models could significantly improve yield forecasting (although past weather data might improve parameter estimation). This variability also imposes limits on the gains from model refinement. A diminishing returns situation is reached quickly, as the “signal to noise ratio” and the improvements in the ratio of forecasting accuracy to precision decrease.

Another characteristic of stand development that limits the usefulness of more complex models is the interdependence of variables, analogous to multicollinearity in linear regression. For example, given two of the variables mean diameter, number of trees, and height, only a narrow range for the third variable is feasible (Decourt 1974, García 1988a). A consequence of this kind of situation is that different hypotheses can “explain” the data nearly equally well. In connection with the models described here this is demonstrated in Figure 9. Figure 9a shows that in the Pumice Plateau data there is a strong relationship between volume and estimated closure (for a similar relationship between crown cover and basal area see figure 1 in Shepherd and Forrest, 1973). Therefore, both the relationship between increment and volume suggested by Langsaeter (Braathe 1957), and that with closure discussed before appear equally reason-

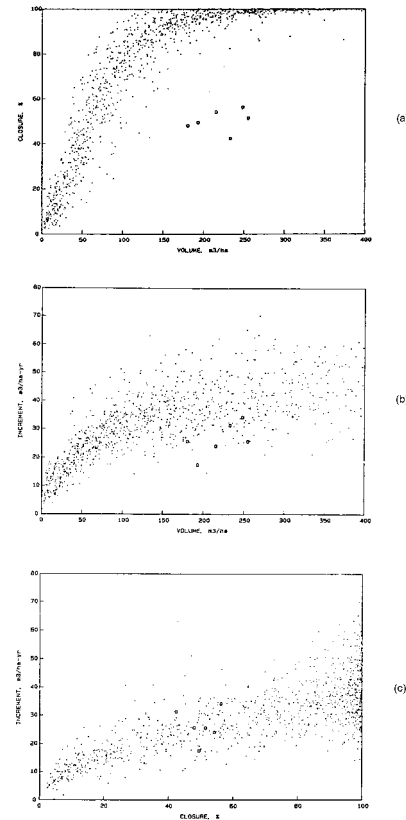


Figure 9: Example of the effect of interrelated variables on hypothesis validation. 1-year gross volume increments, closure estimated by PPM88. Measurements with volumes under 400 cu m/ha. Because of the relation shown in (a), both of the alternative causal relationships suggested in (b) and (c) appear reasonable, except possibly for the “atypical” points represented as circles.

able (Figures 9b and 9c). Only for “atypical” points (plotted as circles) the difference may be important. A similar argument explains why the introduction of closure in our models improves the predictions mainly for the more drastic treatments.

Despite not having a dramatic effect in the presence of other state variables, the use of closure together with a simple expression for the volume increment of closed stands seems promising. It could form the core of more accurate and robust versions of our models. More importantly, a model with few param-

eters would be useful in “data-poor” situations, e.g. to develop models for some of the New Zealand plantations other than radiata pine or Douglas fir. Lastly, it may enable more robust regime evaluations.

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