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ABSTRACT. A number of stand growth models have been developed in New Zealand using relatively sophisticated mathematical and statistical techniques. The models consist of a system of stochastic differential equations, and the parameters are simultaneously estimated by maximum likelihood. Five models have been completed and another two are under development.

INTRODUCTION

A series of stand growth models covering the main commercial forests and forest regions of New Zealand is being completed. A general methodology was developed to facilitate the production and updating of these models.

I give in the next section some background information on characteristics of New Zealand forestry that have influenced our approach. Then the methods and results obtained are briefly described.

More detailed accounts of the methodology have been given by García (1979, 1983, 1984). For other growth modelling efforts in New Zealand see Goulding (1986) and Tennent (1982).

THE FORESTRY CONTEXT

New Zealand has practiced plantation forestry for almost a century. A combination of diminishing native forest resources and pressure from conservation groups has accelerated a shift to man-made production forests. By 1955, wood production from plantations equalled that from the native forests. There are now 1.1 million hectares of plantations, accounting for 95% of the annual production of 10 million cubic meters.

The most successful species has been radiata pine (*Pinus radiata* D. Don), of which there are over 1 million hectares. In a distant second place comes Douglas fir (*Pseudotsuga menziesii* (Mirbel) Franco), at 61 thousand hectares. Radiata pine is the world's most extensively planted exotic softwood, and is characterized by fast growth and a versatile timber. Its predominance in New Zealand has resulted in considerable research effort directed at this species.

The concentration of planting activity in the 1920s and 1930s, and then again after 1960, produced a very uneven age-class distribution. The result is large changes in rotation age and supply conditions over

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time. After a period of abundance, we are currently in a comparatively tight timber supply situation, yet production is projected to double by the end of the century. Clearly, adequate planning under these circumstances requires accurate growth and yield forecasts.

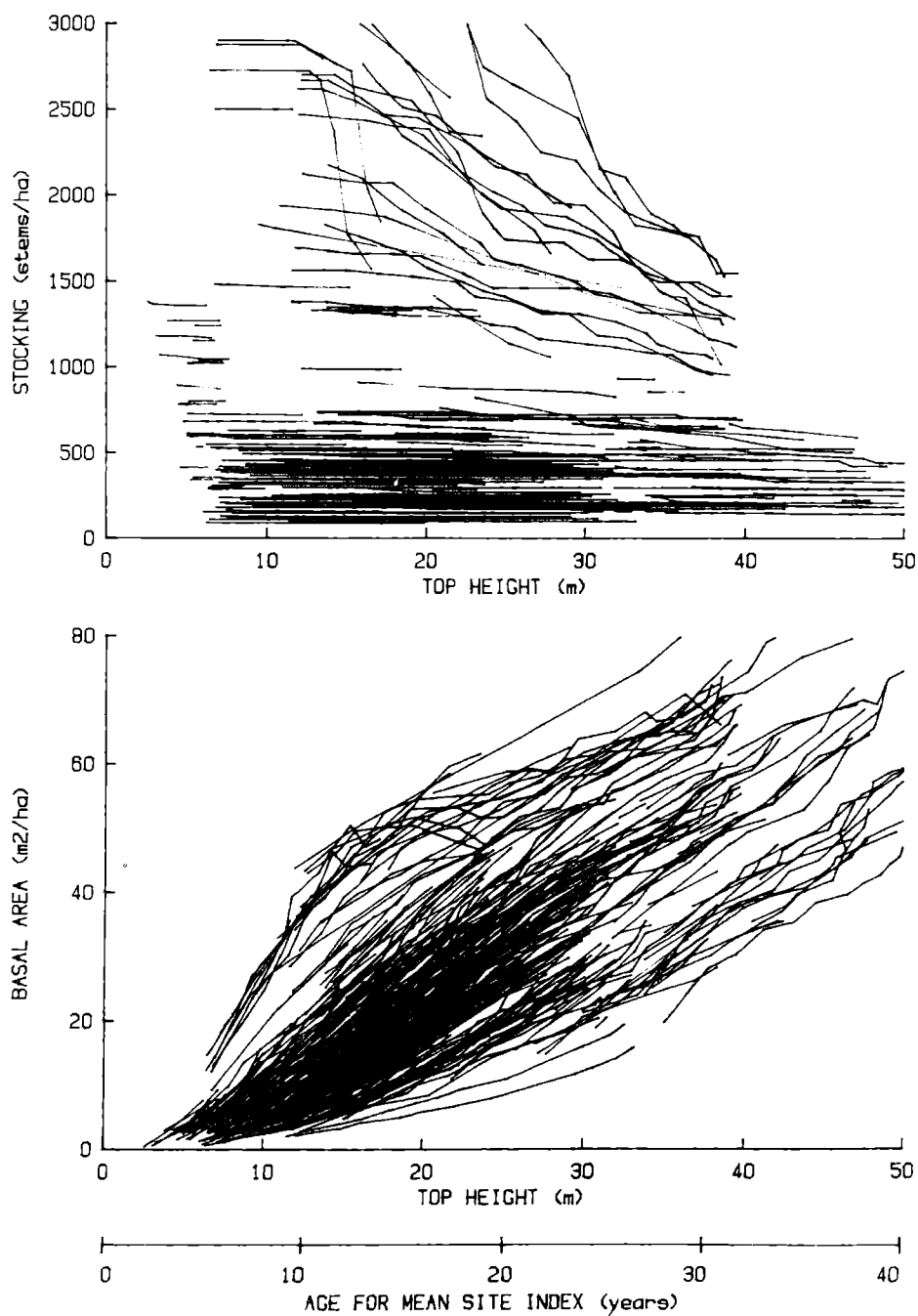


Figure 1. Permanent sample plot data for the Pumice Plateau model. Consecutive measurements between thinnings are joined by lines.

Another factor influencing growth modelling is the high silvical flexibility of radiata pine, and the variety and intensity of silvicultural treatments applied in its management. Figure 1 gives some idea of the range of stockings which currently exist as a result of varying initial spacings and thinning regimes. In the past two decades, silviculturalists have constantly pushed models and data to their limits, by repeatedly advocating lower stocking regimes than were previously considered practicable. Experimentation and data collection has ensured that there is now, at least for some regions, a reasonable data base on the response to the various growing conditions. A recent resurgence of interest in less intensive management emphasizes the importance of sampling and modelling the full range of growing conditions, avoiding the temptation to focus exclusively on current practices.

Although the idea of having one growth model applicable to the whole country is attractive on philosophical grounds, practical considerations indicated the necessity of developing regional models. It was known that there existed important differences, not just in growth rates, but also in growth patterns between regions. For example, there are differences in the shape of site index curves (Burkhart and Tennent, 1977), and radiata pine in Southland produces higher basal areas for a given height than in the central North Island. Other interregional differences in growth patterns have since been found, and are discussed below. Current knowledge is insufficient to allow the parameterization of these differences by continuous environmental variables; only stratification into discrete regions is possible. In addition, with over 7000 permanent sample plots for radiata pine in New Zealand, it was necessary to subdivide the work into more manageable portions. The main radiata pine forests were tentatively classified into 8 groups for growth modelling purposes, based on climate, geology, soils, and availability of data. Once the first set of models for the country is obtained, it will then be possible to assess the desirability of pooling regional data sets.

METHODOLOGY

THE STATE SPACE APPROACH

The growth models are stand-level models, with the state of a stand at any point in time described by a small number of summary variables. These state variables include the net stand basal area, number of stems per hectare, and top height. Some of the models use additional variables representing the degree of canopy closure and/or nutrient levels. Such information is easily obtained by forest managers, and is sufficient for growth prediction in most instances. Other information of interest, such as volumes for various products, or diameter distributions, can be estimated from the values of the state variables. The rates of change of the state between treatments (thinnings, pruning, fertilizing) are modelled by a system of differential equations.

The principles may be explained with the three-dimensional state space depicted in Figure 2. This is a stereoscopic pair in which

measurements from permanent sample plots are shown as points in a space defined by three state variables, basal area, average spacing and top height. Consecutive measurements between thinnings are joined by lines. The basic assumption is that the current location of a stand in this space, plus any subsequent thinnings, completely determine its future development independently of its past history. For any dynamic system, this can always be made true by using a sufficiently detailed state description. An appropriate state description must also allow other required information to be estimated. For example, the stand volume might be estimated by a function of basal area and top height. The slopes of the trajectories at any point in the state space are modelled by differential equations. By integration, the state at any future time can be calculated as a function of the current state. Thinnings cause instantaneous jumps in the state space.

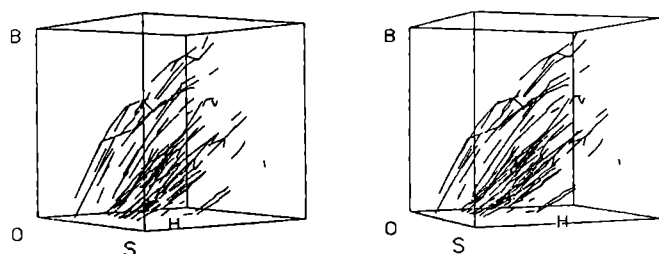


Figure 2. Stereogram showing the Golden Downs data in a three-dimensional state space. B=basal area (0-100 m²/ha), S=average spacing (0-10 m), H=top height (0-50 m).

Incidentally, it can be seen in Figure 2 that the data tend to be concentrated near a surface in the three-dimensional space. A similar effect was noted by Decourt (1974), and may explain why models using two-dimensional state spaces (basal area and height, or basal area and age) often perform reasonably well. We have found, however, substantial improvements in prediction by introducing a third variable.

GROWTH EQUATIONS

The differential equations used can be seen as a multivariate generalization of the Bertalanffy-Richards model. The univariate version of this model, widely used for site index curves, can be written as a linear differential equation with a power transformation of the state variable H, e.g. top height:

$$dH^c/dt = a H^c + b \quad (1)$$

Some of the parameters a, b or c are usually functions of the site

index. For a three-dimensional state vector, for example (B,N,H) consisting of basal area, number of stems per hectare and top height, we use three linear differential equations with power transformations of the form

$$B^{c_1} N^{c_2} H^{c_3} . \quad (2)$$

In general, using standard matrix notation, for an n-dimensional state vector \underline{x} the multivariate version is

$$d\underline{x}^C/dt = A \underline{x}^C + \underline{b} , \quad (3)$$

with \underline{x}^C defined as

$$\underline{x}^C = \exp [C \ln \underline{x}] ,$$

and A, C and \underline{b} being n-dimensional matrices and vectors of parameters.

Notice that many mensurational variables, such as mean diameter, average spacing, and volume estimated by logarithmic volume equations, are of the form (2). Therefore, use of these alternative variables instead of B, N or H would also result in a model of the form (3).

The models developed to date have used three to five state variables, with the site index entering into the equations in various ways. Versions with different subsets of the components of A, \underline{b} and C fixed at zero have been tried. Extensions of (3), where the right-hand-side is multiplied by functions of time or of some of the state variables, have also been used. Some of these variations are discussed in the RESULTS section.

In this approach the fundamental data units are pairs of consecutive measurements. Therefore, the availability of permanent sample plots remeasured over a long period of time, although desirable, is not essential. This makes these methods particularly interesting for situations where long-term experimental data is not available.

ESTIMATION

One of the components of (3) is always of the form (1), representing the height growth and site index curves. The parameters for this equation are estimated first. A stochastic process representing the environmental variation in growth conditions is added to the right-hand-side, and a random variable representing measurement errors is also included. While some of the parameters are common to all plots, others are specific to individual plots (site-dependent). An explicit expression for the likelihood function is obtained, and all the parameters are estimated simultaneously by a direct maximization of this function.

Once the height growth parameters and the site indices for all plots have been estimated, the rest of the parameters in (3) are estimated by a similar procedure. A stochastic process is added to the right-hand-side of (3), but measurement errors are not directly modelled.

Here, after introducing functional dependencies on the site index, the parameters to be estimated are common to all plots. The likelihood function can then be maximized with a general-purpose optimization procedure.

For each region, a number of variations of the general model have been tried, and extensive validation and graphical examination of the residuals has been carried out.

RESULTS

Some information on the models that have been produced, all for radiata pine, is given in the following table.

TABLE 1. Growth models developed with this methodology.

Model	Date finished	Measurement pairs	Sample plots
Golden Downs	1981	339	119
Southland	1982	171	56
Hawkes Bay	1983	634	207
North Island Sands	1984	1100	513
Auckland Clays	1987	510	228
Pumice Plateau	-	2295	297

Most of the work on these models was done by M.E. Lawrence (Southland, Hawkes Bay, Pumice Plateau), A.G. Dunningham (N.I. Sands, Pumice Plateau), and R.G. Shula (Auckland Clays).

The Golden Downs model is the only one that has been formally published to-date (García, 1984). The state variables are basal area, stems per hectare, and top height. Site index enters in a factor multiplying the right-hand-side of (3), i.e., its effect is a change in the time scale.

The Southland data set was used in the initial development of the methodology. Some preliminary results were reported by García (1979). In 1982 the parameters were re-estimated with the same relationships used for Golden Downs. This model is considered as provisional because not all the available data have been used, and only a few variations of the general model have been investigated.

For Hawkes Bay, a reduction in growth was detected immediately following heavy thinnings. A fourth state variable, representing a measure of relative site occupancy or canopy closure was included to account for this effect (García, 1984). Analysis of residuals also showed that the effect of site index on basal area increment was less pronounced than for height increments. A model with different site multipliers for the different equations in (3) accounted satisfactorily for this effect.

For the North Island Sands, no improvement was obtained by introducing a measure of canopy closure and a three-dimensional state vector was

used, as for Golden Downs. The effect of site index for the N. I. Sands, however, is different from that in the other models. The site index curves were found to be anamorphic, i.e., instead of changing the time scale, the site index alters the asymptotes. A model that varies the asymptotes for all the state variables as a function of site index was found to be appropriate.

The Auckland Clays region includes soils with varying degrees of phosphorous deficiency. It is normal management practice to fertilize the forest plantations in this region. A model was developed based on data from plots that had been maintained at an adequate level of P (above 0.11% foliar P). This model also includes a canopy closure variable, as for Hawkes Bay. The site index affects the time scale, as for Golden Downs. Using data from fertilizer experiments, the model was modified to predict the effect of lower phosphorous levels. In this version the foliar P level is included as an fifth state variable, with an associated decay function describing its change over time. The additional variable is included in the other equations via multiplier functions, adjusting the growth in relation to that predicted for the adequately fertilized stands.

The most extensive data set was that for the Pumice Plateau (Figure 1). An interim model with a three-dimensional state vector has been produced, but has been found to overpredict slightly the growth following heavy thinning and pruning. Work is under way to account for these effects. In the other models, any effect of pruning on growth is confounded with other silvicultural treatments, and the lack of data prevents adjustments for it. The Pumice Plateau data, however, includes results from a number of silvicultural experiments and more pruning information. This information is being used through a canopy closure variable affected by both thinning and pruning. Unlike in the previous models, this fourth variable is likely to enter the growth equations in the form of multipliers, allowing for more consistent extrapolations.

Work has started on a radiata pine model for Canterbury. A feature of this model will be the use of data obtained from stem analysis in order to fill gaps in the data base. It is planned to complete an adequate first coverage of the main New Zealand exotic forests with a new model for radiata pine in Southland, and one or more models for Douglas fir.

CONCLUSIONS

The methodology described has proved to be feasible and effective, and has provided a consistent framework for the development of a series of growth models. Excellent predictions have been obtained over a wide range of treatments and growing conditions.

Currently, these methods can not be recommended for general use by the casual growth modeler; the capacity to understand the techniques, and to modify them if necessary, is essential. Also, computing costs may be higher than for some other methods. For those with an adequate level of expertise, however, the efficient use of the data makes this

approach attractive. In most instances, the costs of growth model development are negligible in relation to the costs of data collection and the consequences of prediction errors.

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