FSTY-405 — Math review, practice (2007 Lab 5)

VDYP, functions, and all that

Objectives

To practice typical forestry algebraic manipulations. Understanding functions of one and more variables, and their graphical representations. Mostly for the mathematically impaired. No marks.

Models

For ages up to 120 years, the VDYP yield equation is

$$V = b_0 + b_1 H + b_2 HA + b_3 H^2 C + b_4 AC \ , \tag{1}$$

where V is total volume (m^3/ha), H is top height (m), A is breast-height age (years), and C is crown cover (%). The regression parameters for lodgepole pine are:

i	b_i
0	79.0
1	-10.5
2	0.0628
3	0.0235
4	-0.0830

We will also need the pine site index model (Goudie 1984):

$$
\begin{aligned}
H &= 1.3 + (S - 1.3)\,(b_1 / b_2) \\
b_1 &= 1 + \exp\,[\,7.815 - 1.285 \ln 50 - 1.007 \ln (S - 1.3)] \\
b_2 &= 1 + \exp\,[\,7.815 - 1.285 \ln A - 1.007 \ln (S - 1.3)] \\
ytb &= 5.6 + 42.64 / S
\end{aligned}
$$

where:

H = top height (m)
S = site index (height at 50 years breast-height age)
A = breast-height age (years)
ytb = number of years to reach breast height (1.3 m)
ln = natural logarithm
exp = exponential function to the base e

Note that the b_i in the two models denote different things! What is the meaning of these equations? What is the difference between *variables* and *parameters*?

Functions, substitutions

From (1), it is said that V is a *function* of H, A, and C. In the most general terms, a function is a *mapping*, a correspondence, or a rule that associates to each member of a set (the function's *domain*), a unique member of another set (the *range*). The function can be also thought of as the set of all these pairs of corresponding elements. Most often we deal with functions where the domain and range are sets of numbers, or sets of lists of numbers (vectors). In this example we have a mapping

$$(H, A, C) \longrightarrow V \ .$$

Read the previous paragraph again and try to understand it. What sort of function is jointly defined by the three equations in Goudie's site index model?

Important: It is customary to use the word *function* both for the actual function, as just described, and for *function values*, that is, for elements from the function's range. For instance, in the first sentence of this section. It should be clear from context which is meant.

Write down *one* equation for V as a function of A and C, for site index 20. That is, an expression of the form $V = f(A, C)$. Suggested steps:

1. Substitute $S = 20$, and the b_i into the height equation of Goudie's model. Simplify. This results in an expression for H as a function of A.

2. Factor H in (1), to facilitate the next step. That is, group with parenthesis the terms multiplied by H, so that H appears only twice in the new equation.

3. Substitute for H. Substitute the parameter values. Simplify.

In other words, eliminate first Goudie's b_1 and b_2, and then H.

Think about what you have done.

Solving

Similar to the above, but producing an expression for V as a function of H and C.

First you have to solve the site index model for A as a function of H, and then substitute into the volume equation (i.e., eliminate A).

Graphics

Using gnuplot, produce a 3-D graph of V over A and C. The command for graphing $z = f(x, y)$ is `splot f(x,y)` (for *surface plot*). You can either `set` A and C as the dummy variables, or use the defaults x and y. Set appropriate labels and ranges (`xrange, yrange`). In newer versions of gnuplot you can click and drag on the graph to change the point of view.

Interpret the results. How are surfaces and two-variable functions related?

Graph V over A for several fixed crown cover percentages. Do the same for V over C for given ages. How does this relate to the 3-D graph?

Explore, and think about the graphical representation of functions (and the function representation of curves and surfaces) until you have it clear.

Examples of common non-numeric functions? (a) Area, mapping subsets of the plane into numbers. (b) Derivatives and indefinite integrals, mapping functions into functions (so-called "functionals") (c) Names, mapping people into character strings. (d) . . . ?