FSTY-405 — Lab 5

A simple spatial model

Name: . . . .

Objectives

Understanding basic concepts of spatial tree-level (aka distance-dependent individual-tree) growth models. Spreadsheet practice.

Answer the questions in the spaces provided, attach required printouts, and hand in at the end of the lab for marks. Include your name in the title of all graphs.

Model

For simplicity, we shall look at trees competing in one spatial dimension instead of two (or three). Consider a row of trees, as in a shelter-belt:

```
                    1m
   --o--------o--------o--------o---      . . .
     v1       v2       v3       v4
```
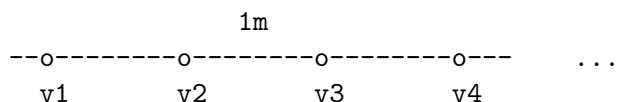
The trees are at 1 metre spacing, and we describe them by their sizes v (*state variables*, e.g., tree volume or biomass).

We assume that a tree growth rate will be proportional to the growing space available to it. We use discrete time:

$$\Delta v \equiv v(t+1) - v(t) \propto \text{growing space}$$

(Notation: $\equiv$ means "equal by definition", $\propto$ is "proportional to").

Assume also that the space between two adjacent trees is partitioned among them, in direct proportion to their sizes (as in a size-weighted APA approach):

```
                         p       q
    ---o-------------o-----+-------o----
       vL            v             vR
```
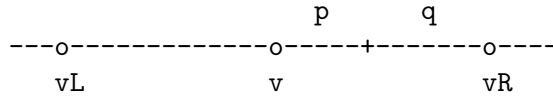
That is, the space between a tree and the one on its right is split into two parts, p and q, proportional to their respective sizes v and v_R. Knowing that $p + q = 1$, we must have:

$$p = \frac{v}{v + v_R} \ , \qquad q = \frac{v_R}{v + v_R} \ .$$

Taking into account both neighbours, the growth rate is then

$$\Delta v = k\big(\frac{v}{v + v_L} + \frac{v}{v + v_R}\big) \ ,$$

where k is some proportionality constant, which "without loss of generality", as mathematicians like to say, we will take as equal to one (this can always be achieved by choosing appropriate measurement units for v and/or t).

Obviously, if we know the tree sizes at time t, the tree size at the next time step can be calculated as

$$\text{Next } v \equiv v(t + 1) = v(t) + \Delta v$$

(*transition function*).

Think about all this, and make sure that you fully understand it before continuing.

We will simulate the growth of these trees, and see the effect of initial size differences.

Spreadsheet implementation

Open Excel. We will simulate 20 trees, across, representing time down the screen:

```
              Tree location (m)
      Time  1  2    · · ·      20
         0
         1
         2
         ⋮
```

Set up appropriate labels, formats, column widths, etc. Try to fit the 20 trees on screen.

Start with $v = 1$ for all trees at time 0. Choose a tree, say tree 5, and enter the formula for calculating its v for $t = 1$. Note that v depends of the previous value for that tree and its two neighbours:
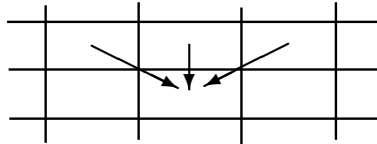
Replicate the formula for all trees at $t = 1$. Examine the results.

What happens at the end of the row (trees 1 and 20)? A common trick to reduce these edge effects is to assume that the same pattern repeats itself: trees on the edge compete with trees on the opposite side. As if the trees were planted on a circle. Fix the formulas assuming that trees 1 and 20 are neighbours.

Replicate the formulas for $t = 2, \ldots, 22$. Add columns for the total volume and for the standard deviation of the tree sizes.

Graph sizes over time (line chart, one line for each tree). Graph sizes over tree location (or tree number). Not too exciting yet!

The standard deviation is a measure of .... Standard deviation at $t = 20$:............... Why?
....

Stop and think.

Varying initial sizes

Choose a tree. Change its size up or down by 10 to 20%. Look at the numbers, graph the results in several ways. What happens? Why?
....
....
Print and attach a graph illustrating your answer.

Now let us use random initial sizes. The Excel `RAND()` function gives a uniformly distributed random number between 0 and 1. Set the initial size of

all trees to `RAND() + 0.5` (random between 0.5 and 1.5). To avoid recalculatation when anything is changed, you can *cut* the row, and reinstate it using *paste special*, choosing to paste values only.

Graph sizes over tree location, and over time. Graph the total and standard deviation.

Standard deviation at $t = 20$:................ What happens to it over time, and why? ....
....
....

Losses, mortality

What happens to growth over time? Is this realistic? Why not?

There should be also losses, independent of growing space. Usually increasing with tree size. Overheads in maintaining branch and root systems (maintenance respiration), foliage and litter production, etc.

Let us subtract then a loss term, say proportional to $\sqrt{v}$:

$$\Delta v = (\frac{v}{v + v_L} + \frac{v}{v + v_R}) - 0.5\sqrt{v}$$

(I have chosen the 0.5 by trial-and-error).

Implement this change in the spreadsheet. Graph the v over time.

Shrinking trees? Can't happen? What happens in real life?

In reality, a tree with a negative (potential) increment cannot sustain itself, and would probably die. Use the Excel `IF()` function to modify the transition function so that if the increment becomes negative the new size is zero (dead).

Examine the behaviour of the system. Compare the sizes of trees having different numbers of dead neighbours.

Attach a graph with all the tree sizes over time.

What happens (in this model) to its growing space when a tree dies? How is it shared among its neighbours?
....
....
....
(hint: look at the model formulation again). Can you think of other ways of modelling that situation?