

An Option Theory Based Yield Curve Model

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Abstract

The Black-Scholes option theory provides a simple analytical model about the yields of corporate bonds. We extend the theory to model the yields of default free government bonds from a simple observation. From the purchasing power perspective, the values of corporate and government bonds follow similar patterns. The option theory based model of the term structure of interest rates explains major empirical patterns on the shapes and dynamics of yield curves. Compared with existing theories on yield curves, this theory provides a simple analytical theory without additional assumptions about risk, liquidity and preference. It greatly simplifies the understanding and teaching of yield curves.

Keywords: option, yield curve model

1. Introduction

Option theory has been applied to understand the yield of corporate bonds since the very first paper by Black and Scholes (1973). We will show that the same theory can be extended to understand government bond yield curves. Governments, like corporations, can face financial difficulties. While governments normally do not default on their bond obligations, the necessary monetary policies that governments adopt in times of financial difficulty often devalue their currencies. From the perspective of purchasing power, government bonds share the same types of risk as the corporate bonds. This simple understanding enables us to apply the Black-Schoels theory of corporate bond to analyze government bond yield curves.

The resulting theory is a simple analytical model of three parameters representing leverage, volatility, and general level of yield. It explain the three important empirical facts of the term structure of interest rates (Mishkin and Serletis, 2008). Among existing theories on yield curves, some are qualitative and others are quantitative. Compared with the qualitative theories on yield curves, such as expectation theory, segmented market theory, liquidity premium theory and preferred habitat theory, the option theory based model provides a simple analytical theory without additional assumptions about risk, liquidity and preference. Research on quantitative yield curve models is very active. Some recent examples include Collin-Dufresne, Goldstein and Jones (2008) and Ang, Bekaert and Wei (2008). Compared with sophisticated mathematical and econometrical models in these quantitative yield curve theories, the option theory based model is much

simpler, and yet explains the main empirical facts about yield curves. Overall, this theory greatly simplifies the understanding and teaching of yield curves.

The rest of the paper is structured as follows. Section two reproduces the Black-Scholes theory of corporate bond yield. Then we extend the idea to understand the yield curves of default free government bond. In section three, we show that the option theory based yield curve model explains the three important empirical facts documented in the literature. Section four concludes.

2. The Option Theory Based Model of Bond Yields

It is often assumed that the asset of a company follows lognormal process, that is

$$\frac{dS}{S} = \mu dt + \sigma dz$$

where S is the asset of a company and μ and σ are the growth rate and uncertainty of the asset respectively. The asset of a firm is financed by equity and debt, that is

$$S = equity + debt$$

To simplify discussion, we further assume the debt is a zero coupon bond which matures in year T . The amount of repayment of the debt is K at the end of the maturity. At time T , if the asset value is higher than K , the equity holder will payoff the debt. If the asset value is lower than K , the equity holder will declare bankruptcy and leave the asset to the debt

holders. In effect, the debt holders write a put option with strike price K to equity holders. Therefore the value of debt is

$$debt = Ke^{-rT} - P(S, K) \quad (1)$$

where r is the risk free interest rate and P is the put option whose value can be calculated from the Black-Scholes option formula. From the above equation, we can calculate K . The bond yield, R , is determined by the equation

$$debt = Ke^{-RT} \quad (2)$$

This derivation is based on Black and Scholes (1973).

The above analysis concerns about yields of corporate bonds, which have default possibilities. The same analysis can be applied to default free government bonds. Government income and expenditure fluctuate with economic conditions, demographics, political policies and tax codes, just like ordinary companies. While a government can take various measures to avoid default on its debt obligations, each measure has its own cost that may affect the value of the currency. For example, when government income is low, it may have to print money to finance debt payment, which reduces the purchasing power of its currency. So while governments usually don't default on their debt payment, the real value of government bond may behave like that of corporate bond. Only the volatility of government bond is lower than that of corporate bond since incomes from a

government are more diversified than a corporation and a government is more powerful in generating revenues than a corporation. Therefore the same analysis can be applied to understand government bond yield curve as well. Whether such a theory is valuable depends on its ability to explain major empirical facts about the term structure of interest rates. In the next section, we will explore the properties of this yield curve model.

3. Properties of the Option Theory Based Yield Curve Model

“A good theory of the term structure of interest rates must explain the following three important empirical facts:

1. ... interest rates on bonds of different maturities move together over time.
2. When short-term interest rates are low, yield curve are more likely to have an upward slope; when short-term interest rates are high, yield curves are more likely to slope downward and be inverted.
3. Yield curves almost always slope upward.” (Mishkin and Serletis, 2008, p. 126)

We will investigate how well this yield curve model explains the empirical facts. First, we calculate how bond yields change with the maturity. We change the maturity of the bond and keep all other parameters constant. The yield curve shown in Figure 1 is representative of the relation between years of maturity of bonds and their yields for most parameter values of volatility and leverage. It shows that yield curves almost always

slope upward. This explains the third empirical fact. Next we will investigate when yield curves slope downward and are inverted

Second, we investigate how leverage ratios affect the shape of yield curves. We change the leverage ratio and keep all other parameters constant. Calculation shows that when leverage is extremely high, short-term interest rates are very high and yield curves slope downward and are inverted. Figure 2 presents three yield curves where leverages are extremely high, high and low.

Third, we investigate how the change of each parameter affects the shape of yield curve as a whole. Figure 3 draws yield curves for companies or countries with different leverage ratios. Highly leveraged company or country has higher yield. Figure 4 draws yield curves for companies or countries with different levels of volatility. Higher volatility company or country has higher yield on its bond. We also investigate how the change of r affects yields. We find that the change of r will cause yields to shift the same amount across all maturities. These patterns show that interest rates on bonds of different maturities move together over time.

From the above discussion, the option theory based yield curve model does explain the three important empirical facts.

Compared with existing theories on yield curves, the option theory based model provides a simple analytical theory without additional assumptions about risk, liquidity and

preference. For example, preferred habitat theory, one of the leading theories on yield curve structures,

Assumes that investors have a preference for bonds of one maturity over another, a particular bond maturity (preferred habitat) in which they prefer to invest. Because they prefer bonds of one maturity over another, they will be willing to buy bonds that do not have the preferred maturity only if they earn a somewhat higher expected return. Because investors are likely to prefer the habitat of short-term bonds to that of longer-term bonds, they are willing to hold long-term bonds only if they have higher expected returns. (Mishkin and Serletis, 2008, p. 134)

The preferred habitat theory does not offer any quantifiable statement about the yield. On the other hand, the option based theory provides a precise prediction about how leverage ratio and volatility affect the shape of yield curves. Therefore the option based yield curve is empirically testable while existing theories on yield curves are not subject to rigorous empirical tests.

The difference between the existing yield curve theories and the option based yield curve theory reflects a deeper difference between research methodologies in economics and finance. This is best captured by Mehrling (2005) in his study on the origin of the option theory.

Coming from economics, it was natural for Samuelson, and for Merton following him, to think of the option pricing problem from the point of view of the

individual investor considering the range and probability of values that the option might have upon maturity, and then discounting those future values back to the present. From this point of view, it seems obvious that the current price of the option must depend on the investor's attitude toward risk. Even more, since the option is more risky than the stock, it seems intuitive that, if the investor is to hold both the option and the stock, the expected return on the option must be higher than the expected return on the stock. How much higher must depend on both the investor's attitude toward risk and the riskiness of the option, and (just to make things harder) the riskiness of the option changes with the stock price. It seems like a complicated problem.

By contrast, coming from Treynor's CAPM, it was natural for Black to think of the option pricing problem as essentially a matter of calculating exposure to market risk at a moment in time. And it was furthermore natural for him to proceed, following the method of Treynor, by writing down a differential equation describing how the value of the option changes over time. Black's preferred CAPM approach to the problem appears in the published 1973 Black-Scholes article under the heading "An Alternative Derivation," so it needs to be emphasized that this "alternative" was in fact the key that he used to unlock the problem in the first place. In 1969 Black was applying CAPM not only to options but also to lifetime investment strategy, to money and to business cycles. To understand how he was able to crack open the problem that had so far defeated everyone else, we must start where he started. (Mehrling, 2005, p. 128)

This work provides another example that a preference-free methodology offers more concrete results than preference based theory.

4. Concluding Remarks

This work extends the option theory based yield curve model from bonds with default risk to default free bonds. The resulting analytical theory is highly consistent with important empirical patterns documented in the literature. Given the simplicity of this conceptual extension, the effectiveness of this model in describing the empirical facts is somewhat surprising. Being an analytical theory, the model also generates some testable predictions beyond the three important empirical facts about the term structure of interest rates. Their empirical validity is left to future research.

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Figure captions

Figure 1: Relation between years of maturity of loans and yield. Parameters: equity, 1 million; debt, 2 million; risk free rate, 7%; volatility, 25%.

Figure 2: Extreme leverage and yield curves: Parameters: risk free rate, 5%; volatility, 25%, debt, 2 million, equity, 0.1 million, 0.3 million and 1 million for three differently leveraged companies.

Figure 3: Leverage and yield curve. Parameters: risk free rate, 7%; volatility, 25%, asset, 3 million, debt, 1.5 million for low leverage company and 2 million for high leverage company.

Figure 4: Volatility and yield curve. Parameters: risk free rate, 7%; asset, 3 million, debt, 2 million; volatility, 15%, for low volatility company, 25% for high volatility company.







