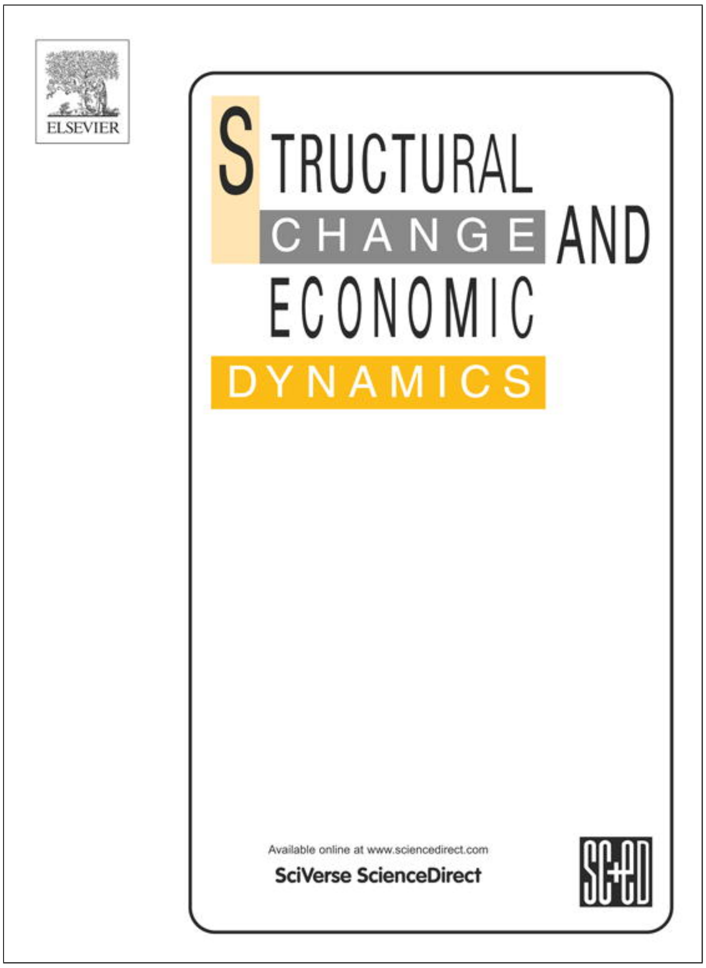
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The nature of discounting☆

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ABSTRACT

From monetary policies to the climate change problem, from the burden of private credit card debts to the evaluation of public projects, discount rate is the central issue, yet there is little clear understanding about the nature of discounting. In this paper, applying a newly developed production theory, we discuss how discount rate is related to other factors in social systems, such as risk, duration of production, fixed cost in production and market size. The relations among different factors in a social system put constraints on the ranges of discount rate that are viable in particular environments. Our findings have strong policy implications. In a world of increasing cost of extracting natural resources, the continuation of low discount rate policy will generate wide gyration of social systems that we have witnessed in recent years.

1. Introduction

From monetary policies to the climate change problem, from the burden of private credit card debts to the evaluation of public projects, discount rate is the central issue. Much effort has been made to understand the tradeoffs among investment and consumption behaviors at different time intervals, yet there is little clear understanding about the nature of discounting. Martin Weitzman, the world's leading expert on the social discount rate, commented:

> The concept of a "discount rate" is central to economic analysis . . . Because of this centrality, the choice of an appropriate discount rate is one of the most critical problems in all of economics. And yet, to be perfectly honest, a great deal of uncertainty beclouds this very issue. . . . The most critical single problem with discounting future benefits and cost is that no consensus now exists, or for that matter has ever existed, about what actual rate of interest to use. (Weitzman, 2001, p. 260)

The main problem in the theory of discounting, as pointed out by Weitzman, is that "an economist who knows the literature well" is "able to justify *any* reasonable social discount rate by some internally consistent story". In this paper, we will discuss how the discount rate is related to other factors in biological and social production systems, such as risk, duration of production or life span, fixed cost in production and size of market. The relations among different factors in a production system will put constraints on the ranges of discount rate that are viable in particular environments. These constraints will help us understand how discounting should be applied in different situations. But before our analysis, we would like to list some puzzles related to discount rates.

The borrowing rates for banks are very low. But the credit card interest rates that banks can charge their customers are very high. How can the large interest rate differential be maintained over a long time? From another perspective, individuals can obtain a line of credit at a much

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lower interest rate than the credit card interest rate. Why do so many people still maintain a large amount of credit card debt, without replacing it with a line of credit?

Most economists are staunch proponents of the efficiency of markets, especially when the market is very liquid and transparent. The short term money market is among the most liquid and transparent markets in the world. Yet, most economists support that the short term discount rate, possibly the most important factor affecting economic performance, should be determined by a small group of "independent" professionals from central banks. Does that mean the market is only capable of being efficient on minor issues and not on major issues?

In general, yield curves slope upward. Loans with longer maturity pay higher interest rates than loans with shorter maturity. At the same time, empirical evidence suggests that humans discount the long term future at lower rates than the short term future (Ainslie, 1992; Berns et al., 2007). Many policy papers also advocate discounting long term projects at lower rates than short term projects (Weitzman, 2001; Newell and Pizer, 2003). Why do market discount rates and psychological and policy discount rates move in different directions in relation to the increase in project duration?

Government bonds are classified as risk free securities and cash flows from government bonds are discounted at the risk free rate, the lowest possible rate. Cash flows from physical commodities are discounted at a higher rate to reflect their risk. On the other hand, for most of the time in human history, currencies were represented by physical commodities or pegged to physical commodities. It is often under financial constraint that governments are forced to abandon the link between currency and physical commodities. Physical commodities are seen as anchors for paper currencies and hence, are less risky than government securities. If this is true, why should incomes from physical assets always be discounted at higher rates than those of government securities?

Biological data contain much richer samples over a much longer time horizon than that of human societies. We will borrow insights from ecological and evolutionary studies to understand how discount rates are related to other factors in economic activities. First, discount rates are closely related to the duration of production or lifespan of organisms. In steady states, the rate of reproduction is equal to the rate of death. Therefore, in biological literature, the discount rate is often set to be equal to the rate of reproduction (Stearns, 1992). Bacteria can reproduce themselves in just thirty minutes. Humans can reproduce only after ten or more years old. Hence, discount rates for bacteria are measured in hours while discount rates for humans are measured in years. In general, organisms with longer life spans have lower fertility rates and hence they have lower discount rates (Lane, 2002). Many empirical studies have documented that humans, as well as other animals, often discount long duration events at lower rates than short duration events (Frederick et al., 2004). This pattern is called hyperbolic discounting.

Second, discount rates are closely related to risk. In general, organisms facing a high level of environmental risk, such as predation, have higher discount rates (Stearns, 1992). In economic theory, the standard quantitative model on the relation between discounting and risk is the capital asset pricing model (CAPM). In the CAPM world, assets are assumed to be infinitely diversifiable and highly liquid. While these assumptions are good approximations of reality in some cases, they are not in many of the most important investment decisions. For example, a four year university education provides you with one bachelor's degree. But one cannot diversify education by studying four different subjects and getting four one-fourth degrees. In finance textbooks, many methods are offered to adjust discount rates in individual projects (Damodaran, 2001). While these adjustments provide flexibilities in determining discount rates, they are often less systematic and ad hoc. Since many risks are non-diversifiable, the measurement of risk will include risk to individual projects as well as systematic risk (Mehrling, 2005).

Third, discount rates are closely related to fixed cost in production. When we have invested a large sum on something, we will take extra care that the value of investment depreciates slowly. From another perspective, in a low interest rate environment, the cost of borrowing is low. Investments with higher fixed cost will benefit. Investments with lower fixed costs are less sensitive to the level of discount rates. Much empirical evidence suggests that the "rate of temporal discounting decreases with the amount of reward" (Thaler, 1981; Green et al., 1997).

The above discussion shows that discount rates, duration of production, risk and fixed costs are highly related. Lower discount rates are positively correlated with long duration, low risk and high fixed costs. High discount rates are positively correlated with short duration, high risk and low fixed cost. These results are consistent with the classification of organisms in ecological studies. Organisms are often classified as K strategists and r strategists. K strategists often have high fixed costs, long duration and low discount rates. They are highly competitive in stable environment. r strategists often have low fixed costs, short durations and high discount rates. They thrive in volatile environment (Colinvaux, 1986; Stearns, 1992). Our results about the relation between discount rate and other factors are very similar to ones that are obtained in the study of mind of humans and other animals (Ainslie, 1992). This shows that the human and animal mind, a product of evolution, understands the important relations in life very well.

The relation among various factors in an economic system can be understood more precisely with an analytical theory. Recently, a mathematical theory of economics of human society and life systems has been derived from the laws of statistical thermodynamics (Chen, 2005; Chen and Galbraith, 2011, 2012a,b). The main result is a formula of variable cost as a mathematical function of discount rate, product value, fixed cost, risk and duration of production. This formula, together with fixed cost and volume of output, allows us to compute and analyze the returns and profits of different production systems under various kinds of environment in a simple and systematic way. The calculated results will provide us with a more precise understanding of the relation among discount rate and other factors in an economic or biological system.

In Sections 2–4, we will present an analytical theory of production and show that it gives a more precise understanding of how discount rate is related to other factors in economic activities. In Section 5, we will discuss why the huge differential between the rates banks can charge on credit card debts and the rates of banks' borrowing can persist indefinitely. The discount rates many people are used to reflect our evolutionary past; the discount rates governments and financial industry try to maintain reflect the current economic conditions. The huge differential between credit card interest rate and borrowing rate of the banks reflects the huge difference between the modern environment and the environment of our evolutionary past.

In Section 6, we will discuss how policies on discount rate affect economic growth, inflation and risks to our economic system. Low discount rates lower the cost of financing. This helps stimulate economic growth. At the same time, expansion of the economy tends to strain the availability of raw materials, which often puts an upward pressure on commodity prices. In the past, the abundance of cheap oil ensured that inflation pressure was mild most of the time. However, geological studies show that the extraction cost of petroleum will increase steadily in the future (Campbell and Laherrere, 1998). With increasing inflation pressure, low interest rate policy will stimulate less and less economic activities and more and more speculative activities. The wide gyration of economic activities we have experienced in the past several years under the mainly low interest rate environment will only be a mild prelude to a much more violent future if the low interest rate policy persists into the future.

In Section 7, we will discuss discount rate policies for projects with long term impacts. It is often argued that such projects should be discounted at very low rates. Bacteria have had a strong impact over the earth's environment for billions of years, yet they have very high discount rates. This shows that systems with long term impacts need not be discounted at low rates.

In Section 8, we will discuss the discounting of physical and financial assets. From the purchasing power perspective, investing in physical assets, such as hydro dams, is not necessarily riskier than investing in financial assets, such as nominally risk free government bonds. Therefore, for private households and public institutions, which are mainly concerned about the stability of social functions, it is justifiable to discount future benefits from a physical investment at a lower rate than the nominally risk free rate.

2. Relations between fixed and variable costs

Among the various relations of different factors, the relation between fixed cost and variable cost is probably the most important. We will elaborate on this relation further before deriving the formal theory. People observed that useful energy comes from the differential or gradient between two parts of a system. In general, the higher the differential between two parts of a system, the more efficient the work becomes. At the same time, it is more difficult to maintain a system with high differential. In other words, a lower variable cost system requires higher fixed cost to maintain it. This is a general principle. We can list several familiar examples from physics and engineering, biology and economics.

In an internal combustion engine, the higher the temperature differential between the combustion chamber and the environment, the higher the efficiency in transforming heat into work. This is the famed Carnot's Principle, the foundation of thermodynamics. At the same time, it is more expensive to build a combustion chamber that can withstand higher temperature and pressure. Diesel burns at higher temperature than gasoline. This is why the energy efficiency of diesel engine is higher than that of gasoline engine and the cost of building a diesel engine is higher as well. In electricity transmission, higher voltage will lower heat loss. At the same time, higher voltage transmission systems are more expensive to build and maintain because the distance from the line to the ground has to be longer to reduce the risk of electric shock. The differential of water levels inside and outside a hydro dam generates electricity. The higher the hydro dam, the more electricity can be generated. At the same time, a higher hydro dam is more costly to build and maintain. A TV with remote control is easier to operate than one without remote control. But to keep the remote control active, electricity is consumed 24 hours a day in a TV.

Warm blooded animal can generate high energy output longer than cold blooded animals because their body temperature is maintained at high levels to ensure fast biochemical reactions. But the basic metabolism rates of warm blooded animals are much higher than the cold blooded animals. This production theory provides a clear understanding to the patterns of temperature regulation, which has not been fully understood in physiological research.

> The fact is that we do not fully understand the advantage of any given body temperature. In any event, it would be a mistake to interpret a low body temperature as a sign of "primitive" and thus inadequate temperature regulation. It has been said that the egg-laying echidna is halfway to being a cold-blooded animal and is unable to regulate its body temperature adequately. In fact, the echidna is an excellent temperature regulator and can maintain its core temperature over a wide range of ambient temperature down to freezing or below, although it has poor tolerance to high temperature. (Schmidt-Nielson, 1997, p. 245)

From the new production theory, higher temperature represents higher fixed cost and low variable cost. Whether a system will evolve toward higher temperature is determined by whether such evolution will help improve return from such a change. Specifically, how much the increased temperature will help increase efficiency in catching prey and avoiding predators. For mammals of low temperature, their prey may be insects or other animals that do not run very fast. So animals with low temperature (30 °C) are fast enough to catch slow moving prey. Avoiding predator faster may not fully compensate the cost of increasing body temperature. Therefore, our theory turns the discussion into a problem of quantitative measurement.

The tradeoff between fixed cost and variable cost is also universal in economic activities. Shops located near high traffic flows generate high sales volume per unit time. But the rental costs in such locations are also higher. Well trained employees work more efficiently. But employee training is costly. People with higher education levels on average command higher income. But education takes time, effort and money. The tradeoff between lower variable cost and higher fixed cost is often not explicitly discussed in the same literature and is often not considered in policy issues. For example, electricity generated from solar panel is considered clean energy because solar panel does not need fuels that will cause environmental problems. But the manufacturing of solar panels is highly resource intensive and highly pollutive. However, the pollution from manufacturing solar panels, the fixed cost part of the solar electricity, is rarely mentioned in policy discussion. While it is in the interest of the promoters of "clean" energy and "renewable" energy to avoid discussing such issues, a good economic theory should provide guidance to understand the big pictures.

From the production theory, it can be calculated that when the fixed cost is zero, the variable cost is equal to the product value, and profit is zero. This means that any organisms or organizations have to make a fixed investment before earning a positive return. Stiglitz made similar observation:

> Timing (and sequencing) is everything. These are not just issues of pragmatics, of "implementation": these are issues of principle. . . .Trade liberalization is supposed to enhance a country's income by forcing resources to move from less productive uses to more productive uses; as economists would say, utilizing comparative advantage. But moving resources from low-productivity uses to zero productivity does not enrich a country. It is easy to destroy jobs, and this is often the immediate impact of trade liberalization, as inefficient industries closed down under pressure from international competition. IMF ideology holds that new, more productive jobs will be created as the old, inefficient jobs that have been created behind protectionist walls are eliminated. But that is simply not the case . . .. It takes capital and entrepreneurship to create new firms and jobs. (Stiglitz, 2002, p. 60)

The determination of the proper level of fixed cost and variable cost to attain high level of return under various environments will be jointly affected by other factors. We will discuss in greater detail after deriving the mathematical theory of production.

3. A mathematical theory of production observing physical and economic principles

We start the investigation by asking: What are the most fundamental properties of organisms and organizations? How do we represent these fundamental properties in a mathematical theory? First, organisms and organizations need to obtain resources from the environment to compensate for the continuous diffusion of resources required to maintain various functions. This fundamental property can be represented mathematically by lognormal processes, which contain both a growth term and a dissipation term.

Suppose S represents the amount of resources accumulated by an organism or the unit price of a commodity, r, the rate of resource extraction or the expected rate of change of price and σ, the rate of diffusion of resources or the rate of volatility of price change. Then the process of S can be represented by the lognormal process

$$\frac{dS}{S} = r\,dt + \sigma\,dz. \tag{1}$$

where $dz = \varepsilon\sqrt{dt}$, $\varepsilon \in N(0, 1)$ is a random variable with standard Gaussian distribution.

The process (1) is a stochastic process. However, most of the time, we observe or sense the average impacts from stochastic processes. For example, the movements of individual gas molecules are very volatile. But the atmosphere, which is densely populated with gas molecules, produces a stable pressure on any surface. We usually study the overall impacts of stochastic processes by looking for the averages of these stochastic variables and their functions. These investigations often transform stochastic processes into their corresponding deterministic equations. For example, heat is a random movement of molecules. Yet the heat process is often studied by using heat equations, a type of deterministic partial differential equation.

Feynman (1948) developed a method of averaging stochastic processes under very general conditions, which is usually called path integral. Kac (1951) extended Feynman's method into a mapping between stochastic processes and partial differential equations, which was later known as the Feynman–Kac formula. Despite its highly technical nature, the Feynman–Kac formula is a very general result and has proven to be extremely useful in many different fields (Kac, 1985). In particular, the Feynman–Kac formula has recently been widely used in research in finance. It was even suggested that "Feynman could be claimed as the father of financial economics" (Dixit and Pindyck, 1994, p. 123).

According to the Feynman–Kac formula (Øksendal, 1998, p. 135), if

$$C(t, S) = e^{-qt}E^S(f(S_t)) \tag{2}$$

is the expected value of a function of S at time t discounted at the rate q, then $C(t,S)$ satisfies the following equation

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} - qC \tag{3}$$

with

$$C(0, S) = f(S) \tag{4}$$

It should be noted that many functions of S satisfy Eq. (3). The specific property of a particular function is determined by the initial condition (4). This is similar to Black–Scholes option theory. The Black–Scholes equation is satisfied by any derivative securities. It is the end condition at contract maturity that determines the specific property of a particular derivative security.

Second, for an organism or an organization to be viable, the total cost of extracting resources has to be less than the amount of resources extracted, or the total cost of operation has to be less than the total revenue. Costs include fixed cost and variable cost. In general, production factors that last for a long time, such as capital equipment, are considered fixed cost while production factors that last for a short time, such as raw materials, are considered variable costs. If employees are on long term contracts, they may be better understood as fixed costs, although in the literature, they are usually classified as variable costs. Typically, a lower variable cost system requires a larger investment in fixed costs, though the converse is not necessarily true. Organisms and organizations can adjust their level of fixed and variable costs to achieve high level of return on their investment. Intuitively, in a large and stable market, firms will invest heavily in fixed cost to reduce variable cost, thus achieving a higher level of economy of scale. In a small or volatile market, firms will invest less in fixed cost to maintain a high level of flexibility. In the following, we will examine the relation between fixed cost and variable cost in a very simple project.

Suppose there is a project with a duration that is infinitesimally small. It only has enough time to produce one unit of product. If the fixed cost is lower than the value of the product, in order to avoid arbitrage opportunity, the variable cost should be the difference between the value of the product and the fixed cost. If the fixed cost is higher than the value of this product, there should be no extra variable cost needed for the product. Mathematically, the relation between fixed cost, variable cost and the value of product in this case is the following:

$$C = \max(S - K, 0) \tag{5}$$

where S is the value of the product, C is the variable cost and K is the fixed cost of the project. When the duration of a project is of a finite value T, relation (5) can be extended into

$$C(0, S) = \max(S - K, 0) \tag{6}$$

as the initial condition for Eq. (3). Eq. (3) with initial condition (6) can be solved to obtain

$$C = Se^{(r-q)T}N(d_1) - Ke^{-qT}N(d_2) \tag{7}$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function $N(x)$ is the cumulative probability distribution function for a standardized normal random variable. From (6), the solution of Eq. (3) can be interpreted as the variable cost of the project. However, we will investigate shortly whether the function represented in formula (7) has common properties of variable costs. For a given investment problem, different parties may select different discount rates. To simplify our investigation, in this paper we will make the discount rate equal to the expected rate of growth. This is to set

$$q = r \tag{8}$$

This choice of discount rate can be understood from two perspectives. First, fast growing organisms also have a high probability of death. In a steady state, the growth rate has to be equal to the death rate. In the biological literature, the discount rate is usually set equal to the growth rate (Stearns, 1992). Similarly, in a steady or competitive state, discount rate should equal to the internal rate of return so the net present value of a project will be zero. Second, in option theory, the discount rate is set equal to the risk free rate. The level of risk of an option contract is represented by implied volatility, which does not necessarily equate with past volatility or future expected volatility. Some people do not agree with the economic logic behind the mathematical derivation of Black–Scholes equation that made the risk related discount rate disappear (Treynor, 1996). However, the disappearance of the separate discount rate greatly simplified our understanding of how option values are related to market variables. From both a biological and economic perspective, this choice of discount rate provides a good starting point for further investigation.

With q equals to r, Eq. (3) becomes

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{9}$$

and solution (7) becomes

$$C = SN(d_1) - Ke^{-rT}N(d_2) \tag{10}$$

This takes the same form as the Black–Scholes formula for European call options. But the meanings of the parameters in this theory differ from that in the option theory. Formula (10) provides an analytical formula of variable cost as a function of product value, fixed cost, uncertainty, duration of project and discount rate of a firm. Similar to understanding in physics, the calculated variable cost is the average expected cost (Kiselev et al., 2000).

We will briefly examine the properties of formula (10) as a representation of variable cost. First, the variable cost is always less the value of the product when the fixed cost is positive. No one will invest in a project if the expected variable cost is higher than the product value. Second, when the fixed cost is zero, the expected variable cost is equal to the value of the product. When the fixed cost approaches zero, the expected variable cost will approach the value of the product. This means that businesses need to make a fixed investment before they can expect a profit. Similarly, all organisms need to invest in a fixed structure before they can extract resources profitably. Some do not agree with this statement and provide examples of low fixed cost invest with high profits, such as J. K. Rowling writing Harry Potter books. Our results are about the statistical average. While a small percentage of authors earn high incomes from blockbusters, an average author does not earn a high income. Third, when fixed costs, K, are higher, variable costs, C, are lower. Fourth, for the same amount of the fixed cost, when the duration of a project, T, is longer, the variable cost is higher. This shows that investment value depreciates with time. Fifth, when risk, σ, increases, the variable cost increases. Sixth, when the discount rate becomes lower, the variable cost decreases. This is due to the lower cost of borrowing. All these properties are consistent with our intuitive understanding of and empirical patterns in production processes.

After obtaining the formula for the variable cost in production, we can calculate the expected profit and rate of return of an investment. Suppose the volume of output during the project life is Q, which is bound by production capacity or market size. During the project life, we assume the present value of the product to be S and the variable cost to be C. Then the total present value of the product and the total cost of production are

$$SQ \quad \text{and} \quad CQ + K \tag{11}$$

respectively. The return of this project can be represented by

$$\ln\left(\frac{SQ}{CQ+K}\right) \tag{12}$$

and the net present value of the project by

$$QS - (QC + K) = Q(S - C) - K \tag{13}$$

It is often difficult to define one unit of output precisely. For example, some products, such as goat milk, are infinitely divisible. In practice, S often represents the value of output from a project over one unit of time. But it can be difficult to decide what unit of time is the most natural choice. In biology, there is a large literature on physiological time of animals. This topic is still an active area of research. We hope our theoretical framework will stimulate further discussion on better definition and measurement of unit value of output, fixed cost and variable cost in practice.

If the project lasts for T units of time, the net present value of the project is

$$TS - (TC + K) = T(S - C) - K \tag{14}$$

We will provide an example to calculate net present value of an investment project. Suppose the fixed investment is 4 billion dollars; annual revenue is 2 billion dollar; discount rate is 2% per year; volatility is 32% per year; duration of project is 10 years. From the above data, we can obtain variable cost as 0.49 billion dollar per year from formula (10) and net present value of the project as 11.10 billion dollar from formula (14).

Jack Treynor's comment about Black–Scholes theory provides a relevant background to understand our production theory (Treynor, 1996):

Time has always been a pesky problem for economists, who have dealt with it by

1. Restricting their model to perpetuities (Modigliani and Miller).
2. Focusing on one-period problems (Markowitz's portfolio balancing model).
3. Reducing the dynamic flow of economic events to a static long run and a static short run (Alfred Marshall).

That these pioneers in quantifying the previous unquantifiable ducked the problems is a measure of what Black–Scholes accomplished.

Our production theory, as an extension of the Black–Scholes methodology, can be applied directly to refine the theories of these pioneers. By considering corporate finance problems in a finite time horizon, we are able to provide a more precise understanding of the problems related to capital structure considered by Modigliani and Miller (Chen, 2006a). By working in a continuous time framework instead of a one period framework, we are able to obtain a more refined understanding of relations among risk, discounting and duration of projects. This helps us understand patterns such as hyperbolic discounting (Ainslie, 1992). By identifying long run cost as fixed cost and short run cost as variable cost and establishing their relations, we provide an analytical theory of economic dynamics that was conceived by Alfred Marshall qualitatively.

Soon after Black–Scholes (1973), it became apparent to many researchers that similar approaches may be applied to capital investment. These approaches are generally called the real option theory. The book by Dixit and Pindyck (1994) is the acknowledged classic in real option theory. In that book, many partial differential equations were derived, but no analytical results about the key factors in capital investment were obtained. As a result, the real option theory "either use stylized numerical examples or adopt a purely conceptual approach to describing how option pricing can be used in capital budgeting" (Megginson, 1997, p. 292). In comparison, the production theory presented in this paper provides simple analytical formulas for the key parameters in capital investment. A detailed literature review and comparison between this theory and the real option theory was provided in Chen (2006b).

The results obtained from this analytical theory are highly consistent with the empirical evidences obtained from the vast amount of literature in economics and biology. Furthermore, by putting major factors of production into a compact mathematical model, the theory provides precise insights into the tradeoffs and constraints of various business and evolutionary strategies that are often lost in intuitive thinking. There have been many criticisms of the theoretical foundation of economics based on Gaussian processes. The properties of this analytical theory of economics, based on Gaussian processes, show that it provides a good starting point for further discussion. We will be very happy if our work can stimulate more refined models based on other theoretical foundations, such as fractal geometry.

In the next section, we will provide a systematic analysis of how discount rates affect cost and return of different projects under different kinds of environment.

4. Relation among discount rate and other factors in a production system

4.1. Discount rate and fixed cost

We discuss how the level of fixed cost affects the preference for discount rates. We will calculate how variable costs change with different discount rates. When discount rates are decreased, the variable costs of high fixed cost systems decrease faster than the variable costs of low fixed cost systems (Fig. 1). This indicates that high fixed cost systems have more incentive to maintain low discount rates

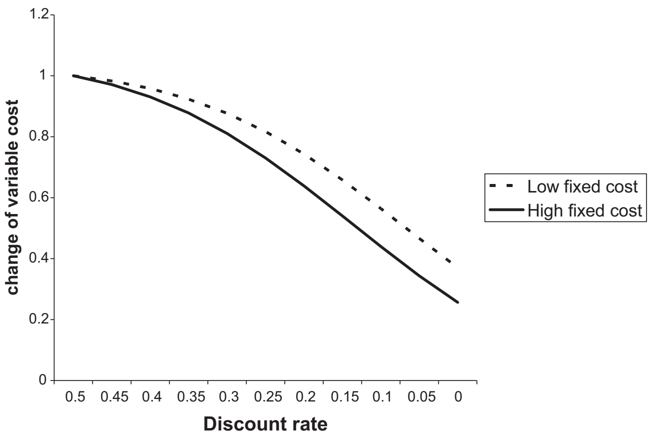

Fig. 1. Discount rate and fixed cost: when discount rates are decreased, variable costs of high fixed cost systems decreases faster than variable costs of low fixed cost systems.

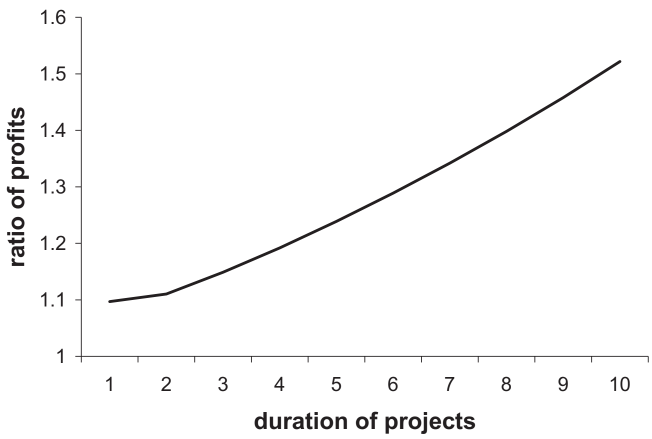

Fig. 2. Discount rate and project duration: the ratios of profits between projects at low and high discount rates at different levels of project duration. Parameters: unit value of product, 1; fixed cost, 1; uncertainty, 55% per annum; annual output, 2; low discount rate, 2% per annum; high discount rate, 8% per annum.

or lending rates. This result helps us understand why prevailing lending rates are different in different areas or at different times.

In poor countries, lending rates are very high; in wealthy countries, lending rates charged by regular financial institutions, other than unsecured personal loans, such as credit card debts, are generally very low. To maintain a low level of lending rates, many credit and legal agencies are needed to inform and enforce, which is very costly. As wealthy countries are of high fixed cost, they are willing to put up the high cost of credit and legal agencies because the efficiency gain from lower lending rate is higher in high fixed cost systems. In the last several hundred years, there has been in general an upward trend in living standard worldwide. There has also been a downward trend in interest rates (Newell and Pizer, 2003).

Empirical investigations show that the human mind intuitively understands the relation between discount rates and levels of assets. In the field of human psychology, there is an empirical regularity called the "magnitude effect" (small outcomes are discounted more than large ones). Most studies that vary outcome size have found that large outcomes are discounted at a lower rate than small ones. In Thaler's (1981) study, respondents were, on average, indifferent between \$15 immediately and \$60 in a year, \$250 immediately and \$350 in a year, and \$3000 immediately and \$4000 in a year, implying discount rates of 139%, 34% and 29%, respectively. Since the human mind is an adaptation to the needs of survival and reproduction, evaluating the relation between discount rate and amount of investment must be a common task in our evolutionary past.

Differences in fixed costs in child bearing between women and men also affect the differences in discount rates between them. Women spend much more effort in child bearing. From our theory, the high fixed investment women put in child bearing would make women's discount rate lower than men's. An informal survey conducted in a classroom survey showed that discount rates of the female students are lower than that of the male students.

Our understanding of discount rate and fixed cost is similar to an earlier work by Ainslie and Herrnstein (1981):

> The biological value of a low discount rate is limited by its requiring the organism to detect which one of all the events occurring over a preceding period of hours or days led to a particular reinforcer. As the discounting rate falls, the informational load increases. Without substantial discounting, a reinforcer would act with nearly full force not only on the behaviors that immediately preceded it, but also on those that had been emitted in past hours or days. The task of factoring out which behaviors had actually led to reward could exceed the information processing capacity of a species.

4.2. Discount rate and project duration

When the discount rate or interest rate becomes lower, the variable cost of a project will decrease and profit will increase. Projects with different lengths of duration will be affected differently by the reduction of discount rates. Fig. 2 presents the ratios of profits between projects at low and high discount rates at different levels of project duration. As project lengths are increased, the ratios increase as well. This indicates that projects with longer duration benefit more from the reduction of discount rates.

Next we calculate the breakeven point of a project with respect to the project duration and the discount rate. Let us assume that project output per unit of time is one. The calculation from formula (14) shows that it requires a lower discount rate to breakeven when the project duration is lengthened. The calculation results are illustrated in Fig. 3. The calculation provides a possible explanation for hyperbolic discounting. Since it takes lower discount rates for long duration projects to breakeven, humans and other animals will discount long duration projects with lower rates.

In the following, we will present more empirical evidence about the inverse relationship between discount rate and the duration of a project or span of life. Fecundity, as well as mortality rate, is proximity for discount rate. Lane (2002) provided a detailed discussion about the tradeoff between longevity and fecundity in biological systems.

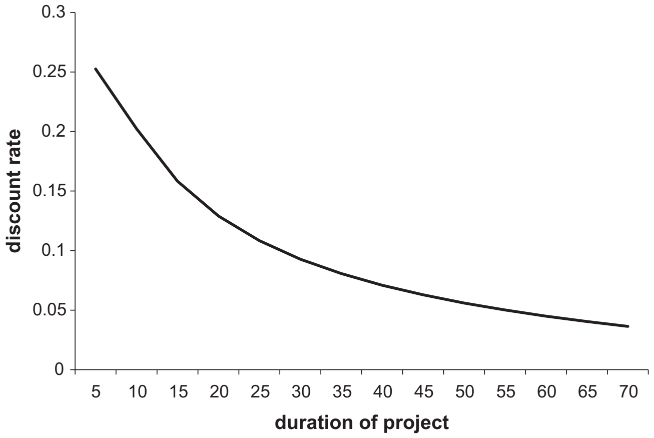

Fig. 3. Required discount rate for the project to break even at different project duration: as project duration increases, required discount rate for the project to break even decreased. This provides a possible explanation for hyperbolic discounting.

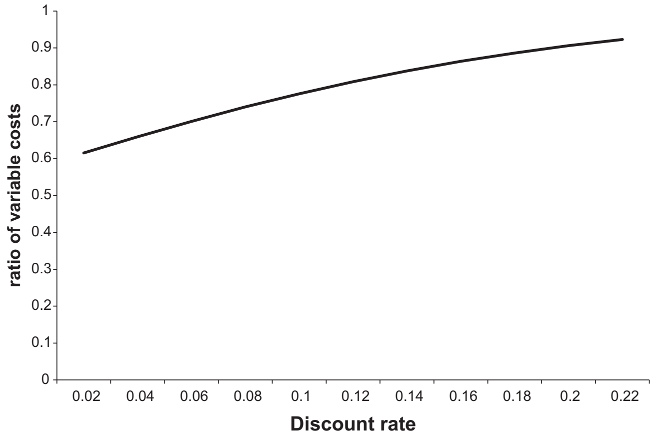

Fig. 4. Discount rate and risk: the ratios of variable costs between low and high levels of uncertainty at different levels of discount rates.

> Notwithstanding difficulties in specifying the maximum lifespan and reproductive potential of animals in the wild, or even in zoos, the answer is an unequivocal yes. With a few exceptions, usually explicable by particular circumstances, there is indeed a strong inverse relationship between fecundity and maximum lifespan. Mice, for example, start breeding at about six weeks old, produce many litters a year, and live for about three years. Domestic cats start breeding at about one year, produce two or three litters annually, and live for about 15 to 20 years. Herbivores usually have one offspring a year and live for 30 to 40 years. The implication is that high fecundity has a cost in terms of survival, and conversely, that investing in long-term survival reduces fecundity.
>
> Do factors that increase lifespan decrease fecundity? There are a number of indications that they do. Calorie restriction, for example, in which animals are fed a balanced low-calorie diet, usually increase maximum life span by 30 to 50 per cent, and lower fecundity during the period of dietary restriction. . . . The rationale in the wild seems clear enough: if food is scarce, unrestrained breeding would threaten the lives of parents as well as offspring. Calorie restriction simulates mild starvation and increases stress-resistance in general. Animals that survive the famine are restored to normal fecundity in times of plenty. But then, if the evolved response to famine is to put life on hold until times of plenty, we would expect to find an inverse relationship between fecundity and survival. (Lane, 2002, p. 229)

He went on to provide many more examples on the inverse relation between longevity and fecundity.

In human society, we often use longevity, or duration of human life as an indicator of the quality of a social environment. At the same time, societies that enjoy a long life span, such as Japan, are often concerned about below replacement fertility. Intuitively, the aging population needs a great amount of resources to maintain their health, which reduces the amount of resources available to support children. Hence, there is a natural tradeoff between longevity and fertility. This result has important policy implications on the balance between resource distribution on longevity and fertility. In a society with below replacement fertility, this poses a great challenge to maintain a sustainable society.

4.3. *Discount rate and risk*

Variable cost is an increasing function of discount rate. When risk is low, variable cost is much lower with a low level of discount rate. When risk is high, variable costs are less sensitive to discount rate. Therefore, it is only important to reduce discount rate in a stable environment. Fig. 4 presents the ratios of variable costs between low and high levels of risk at different levels of discount rates. It shows that reduction of the variable cost is much more significant at a low risk level. This explains why *r* species, which have high discount rates, often thrive in highly uncertain environments. It may show why low interest rates, in a climate of economic crisis, have little effect on the level of perceived profitability and therefore on economic activity. This is called "pushing on a string."

The same idea about the relationship between discount rate and risk was reached earlier. "The same discount curve that is optimally steep for an organism's intelligence in a poorly predictable environment will make him unnecessarily shortsighted in a more predictable environment (Ainslie, 1992, p. 86)".

We have discussed the relations among discount rate and other factors in economic activities. In the next several sections, we will apply these understandings to the questions we raised at the beginning of the paper.

5. Credit card debt rates and bank borrowing rates

Human beings, including the human mind, have evolved over many thousands of generations. Compared with modern society, the lifestyles of hunters, gatherers and farmers are very volatile. Through most of our evolutionary past, both the death rate and the birth rate were very high. The lifespan of our ancestors was much shorter than that of modern humans. Therefore the level of discount rates that our ancestors were familiar with were very high.

Since the human mind is an adaptation to the environment, high discount rates are natural to most people. As a result, many people would accept or at least tolerate a high interest rate charge on their credit card debts.

Modern societies have very high fixed costs. The building and maintenance of roads is very expensive. Education is very costly and the payback period is very long. To reduce the cost of financing, governments need to adopt a policy of low interest rates. To maintain a low level of lending rates, many credit and legal agencies are needed to inform and enforce, which is very costly. As modern societies have high fixed costs, they are willing to put up the high cost of credit and legal agencies because the efficiency gain from lower lending rates is higher in high fixed cost systems. Central banks often adopt low interest rate policies to stimulate economy. Because of the legal, regulatory structures and the monetary policies, borrowing rates by the banks are very low.

In summary, the low borrowing rates of the banks are mainly a result of government policy and regulations; the high lending rates to consumers are mainly a result of people's high discount rate evolved in their mind. In other words, high lending rates in the credit card market are due to market forces and low borrowing rates are due to regulatory forces. The level of profit from credit card issuing firms can be gauged from the huge amount of junk mail one receives from credit card issuing agencies.

There are some more general implications that follow from the above discussion. Despite the existence of many types of borrowing with lower interest rates, interest rates on credit card debt are often higher than 20%. This indicates that the discount rates of individuals are higher than 20%. Due to the hazardous environment we have evolved from, it is natural that we have high discount rates. With a high discount rate, this means that many activities that we engage in are harmful over the long term. This is why in places with lax environmental regulation, economic activities degrade the environment very fast.

We often assume the levels of market interest rate are determined by market forces. Actually, they can be more precisely understood as the combination of regulatory forces and market forces. Market interest rates would be much higher without government regulations and policies.

In the above discussion, we assumed that an individual's discount rate is a single number. In reality, there are many different discount rates in the human mind for different activities. Many psychological problems can best be understood as the conflicting valuations of different activities at different time intervals due to different discount rates (Ainslie, 1992). This also explains the wide range of discount rates of human beings reported in different studies with different methodologies. Since the human brain is a combination of many parts that have evolved at different times, it is natural that different parts have different discount rates. We hypothesize that activities that are controlled by parts of the brain that have evolved at earlier periods have higher discount rates. This hypothesis may be tested empirically.

6. Discount rates, growth, inflation and risk

Economic activities are physical processes. How do monetary policies such as the setting of interest rate affect physical processes? When interest rates are lower, the borrowing costs are lower. This makes it easier for a small number of economic agents to borrow large sums of funds. In other words, in a low interest rate environment, resource uses become more concentrated. We will use an example from biology to illustrate the relation between discount rate and resource concentration.

Mice can start reproduction in several weeks and can multiply many times in one year. Hence, their discount rates are very high. Human beings' reproduction cycles are much longer and hence, their discount rates are much lower. In regard to resource use, people consume much more than mice. As a result, humans are much more competitive than mice in their control and utilization of resources.

From a financial perspective, low discount rates lower the cost of financing. This is especially helpful for high fixed cost projects, which often require substantial financing. Higher fixed cost systems have lower variable cost and hence have more significant scale economy than lower fixed cost systems. When the market size for a product or a service is large and growing, a small number of large projects provide higher returns than a large number of small projects. Financial markets provide an effective way to increase the fixed investment of economic systems and have in the past been instrumental in accelerating economic development.

From the above analysis, we may conclude that while the economic activities are indeed physical activities, discount rates influence how physical systems are structured and hence influence the rate of return of economic activities.

In a growing economy, businesses often anticipate growth. They build up very high fixed cost projects, and with their corresponding low variable costs, take advantage of future large market size. However, high fixed cost systems require a higher levels of output to breakeven. While future demand is expected to be high, the current revenue is often not high enough to support the operation of high fixed cost systems. When there is a large gap between revenue and cost in an economic system, and the capital market is unable to fill the gap, recession occurs. To stimulate demand, governments often lower interest rate to ease borrowing from consumers. Hence, low interest rate policy has been used to stimulate both investment and consumption. Since high fixed cost systems are more competitive in a large and growing economy, and a low interest rate environment helps high fixed cost systems, low interest rate policy has in the past been adopted by most governments.

High fixed cost production systems, with their low variable cost, often reduce the average cost of products in a large and expanding economy. At the same time, the expansion of the economy tends to strain the availability of inputs, which often put an upward pressure on the costs of products. In the past, the abundance of cheap oil ensured that the inflation pressure was mild most of the time.

However, in a now classic paper titled The End of Cheap Oil, Campbell and Laherrere (1998), after carefully examining the oil exploration and production data, concluded "What our society does face, and soon, is the end of the abundant and cheap oil on which all industrial nations depend."

When their paper was published in 1998, oil prices were in the low teens. It would be interesting to examine the events in the world economy after 1998. After the burst of the internet bubble in 2000, interest rates were lowered to stimulate the economy. For several years, the housing market, stock market and commodity market rose continuously, which seemed to suggest that the low interest rate policy worked well in stimulating the economy. However, the rise of commodity prices became unstoppable. Eventually, high commodity prices and the subprime mortgage crisis, both a result of the low interest rate policy, generated the biggest recession since the Great Depression.

Balancing the need for economic growth and a check on inflation has always been a problem in implementing interest rate policies. In the past, governments mostly focused on the goal of economic growth by keeping the interest rate low. Low interest rate environments also generate high rates of inflation in stock and housing prices. But inflation in stock and housing prices is often given a positive interpretation. Inflation in the price of commodities, the raw material of all economic activity, had been kept low for a long time by a steady increase in commodity output. However, with the increasing physical cost of extracting many commodities, it becomes increasingly difficult to keep inflation in commodity prices low while simultaneously demanding a high rate of economic growth.

Low interest rate environments reduce financing cost to build up specialized production systems. It also reduces financing cost in purchasing generic scarce commodities. The scarce commodities could be raw materials, real estate in prime locations, or shares in companies with monopoly powers. Investment in production systems needs highly specialized skills and is very time consuming. The trading of production systems is highly illiquid. The markets for trading of generic commodities are highly liquid. When resources are abundant, there is little inflation pressure on the resource. To make a profit, companies have to make long term investments in manufacturing. That was what happened to the world economy over most of the last several centuries. However, with the population growth and depletion of resources, many resources, such as land in prime locations, mineral reserves and agricultural products, have been under increasing inflation pressure. High inflation pressure in commodities increases investment opportunities in commodities and reduces investment opportunities in manufacturing, which relies on commodities as inputs. As a result, the investment capital has been migrating from specialized and illiquid manufacturing into generic and liquid commodities, real estate and financial assets. Overall, in an age of scarcity of commodities, low interest rate policy will be less and less able to stimulate economic activities and more and more able to stimulate speculative activities.

Discount rate is a reflection of risk. In economic downturns, lenders naturally raise interest rates to compensate for higher risk. However, in economic downturns, central banks, however, often lower interest rate to stimulate economy. Do not central banks and governments, or the general public have to bear the same risks? They do! In response to the bursting of the stock market bubble in early 2000, the central banks lowered interest rates to stimulate the economy. The low interest rate environment inevitably stimulated speculation, especially highly leveraged speculation, which had a higher upside potential. The massive bailout of the major banks and other financial institutions indicated that governments and the general public, like other institutions or individuals, have to bear the same risks for low interest rate loans or guarantees. In the past, risk inherent in low interest policy was compensated for by economic growth generated by low interest policy. We may expect that in the future, with increasing scarcity of resources, risk associated with the low interest rate policy will increase while the stimulus impact will decrease.

Under a low interest rate environment, financial institutions, which incur low borrowing costs, will benefit; depositors, who receive low interest incomes, are harmed. The depositors represent the general public and the financial institutions represent a small minority of people. We may ask why do low interest rate policies prevail for such a long time. This is because in a large and expanding economy, high fixed cost systems provide a higher return than lower fixed cost systems. A low interest rate environment makes high fixed cost systems more competitive. Successful companies create large number of high paying jobs. Part of the large amount of profits brought in by the high fixed cost large firms becomes tax revenues and is redistributed to the general public. Overall, in a large and expanding economy, a low interest rate policy benefits the general public despite their loss as depositors.

With a steady increase in the cost of extracting natural resources, however, steady economic growth has become increasingly difficult. In such economic conditions, the divergence of interest between financial institutions and the general public becomes a very important issue for social stability. In a no growth and declining economy, the problem of income distribution will become very important.

7. Determining the discount rates for long term projects

In general, yield curves slope upward. Loans with longer maturity pay higher interest rates than loans with shorter maturity. This is because longer term loans have to bear higher levels of risk of default. If longer term projects have higher risk, why would the human mind discount long term results at lower rates than short term results? From earlier discussion, we know that discount rate, risk and duration are highly correlated. Only when risk becomes lower, do longer term projects become viable. This means that when we consider longer term projects, our mind implicitly assumes that these projects are of lower risk and apply a lower discount rate accordingly. Therefore, there is no real inconsistency between the direction of movement of market discount rates and psychological discount rates with the increase of project duration.

The topic of climate change has become very prominent in public discussion. Due to the long term impact of climate change policies, it is often argued that projects related to climate change should be discounted at very low rates. Since life emerged on the earth about four billion years ago, organic systems exert a great influence on the earth's environment. It is useful to examine the discount rates of various organisms to understand how discount rates of systems with long term impacts should be determined.

Discount rates vary widely among organisms. Bacteria, one of the oldest forms of life and one of the most important in regulating the global environment, have very high discount rates. Since individual bacteria face a high risk of starvation and predation in their life, their life spans are short and their discount rates are high. The same insight can be applied to determine the discount rates of other projects, including projects related to climate change.

Since considerable funds are at stake in projects and issues related to climate change, there is little appetite for objective investigation in this area. As a result, projects related to climate change often pose high risk for the general public. Corn based ethanol projects, for example, have consumed tremendous amounts of public funding and subsidy. Pimentel and Patzek (2005) reviewed the past research on the production of ethanol from corn and found the following results.

1. The total energy input to produce a liter of ethanol is higher than the energy value of a liter of ethanol. Thus there is a net energy loss in ethanol production from corn.
2. Producing ethanol from corn causes major air and water pollution and degrades the environment.
3. Over 3 billion people in the world are malnourished. Expanding ethanol production will divert corn needed to feed people, and raise serious ethical issues.

Despite the fact that ethanol produced from corn is not a renewable energy, it pollutes environment and raises serious ethical issues, government subsidy on bio-energy has been rapidly expanding over time. As usual, money eclipses science in policy decisions.

The discussion from this and the last sections provides a strong argument that discount rates, and project durations, should be determined by the level of risk. Since a low discount rate environment favors investments with high fixed cost and long duration, policies that lower the discount rate below the level warranted by the level of risk often encourage high fixed cost investments. Such investments promise significant benefit in the distant future, but consume large amounts of resources and are difficult to evaluate by the public. They create an ideal environment for large scale and systematic fraud.

8. Discounting of physical and financial assets

Finance theory classifies government bonds as risk free securities. Cash flows from government bonds are discounted at the risk free rate, the lowest possible rate, while cash flows from other assets are discounted at a higher rate to reflect their risk. However, from the perspective of purchasing power, nominally risk free cash flows may carry higher risk than cash flows from physical resources. Private households and public institutions are mainly concerned about the preservation of purchasing power instead of nominal paper wealth. It is often desirable for these groups to discount income flows from physical resources at lower rates than cash flows from government bonds.

Public investments, such as hydro dams, often have large upfront costs and long durations. When future cash flows are discounted at rates higher than the risk free rate, the net present values for these public projects often become negative. Most public projects are justified on the basis of "externality", which is very ad hoc. However, from a purchasing power perspective, we provide a systematic analysis of risk and discounting that better supports the undertaking of public projects.

Consider the investment analysis for a hydro dam. A hydro dam provides a steady supply of electricity. A well designed hydro dam has a very low engineering risk. While government bonds are nominally risk free, they are subject to inflation risk. Hence, measured from purchasing power, the actual risk of building a hydro dam can be much lower than that of holding government bonds. Specifically, investing in a hydro dam will guarantee a steady supply of electricity over an extended period. Investing the same amount of money in government bonds may not guarantee the purchasing of the same amount of electricity in the future. Therefore, the discount rate for future benefits from a hydro dam should be lower than that for cash flows from government bonds. A lower discount rate should result in the net present values of many public assets becoming positive. Governments need not resort to the use of "externality" to justify such public investments.

The history of the monetary system may help us understand the riskiness of physical assets versus financial assets. For most of human history currencies were represented by physical commodities or pegged to physical commodities. It is often under financial constraint that governments are forced to abandon the link between paper currency and physical commodities. Historically, physical commodities were used as anchors for paper currencies and hence viewed as being less risky than financial assets.

The above discussion does not imply that the short term volatility of prices for particular commodities is lower than that for financial assets. Instead, we have argued that the steady supply of basic physical commodities, such as grains, is essential to the stability of a society. Most governments understand this role. They subsidize domestic agriculture to maintain stable agricultural output and restrict foreign ownership of many natural resource industries.

9. Concluding remarks

Discount rate is related to many other factors in social and biological systems. This makes its understanding very difficult. We often reach contradictory conclusions on discounting when we look at a single factor alone, such as risk or time intervals. In this paper, we showed that many factors in production systems are highly correlated. These correlations put constraints on the choices of parameters of production systems that are viable in particular environments. This understanding clarifies much confusion surrounding many important problems related to discounting.

There are strong policy implications from the findings of this work. A low discount rate policy will help propel a large and growing economy further while a high discount rate policy will cushion a no growth or declining economy. With the steady increase in the physical cost of extracting natural resources, steady economic growth has become increasingly difficult. In such conditions, a gradual increase of discount rate will help stabilize economic outputs. The continuation of low discount rate policy will generate wide gyration of social systems that we have witnessed in the recent years.

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